

## Answers to Selected Problems

### Problem 1.11.

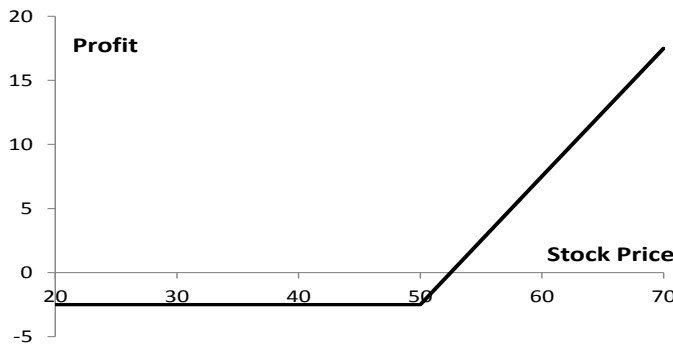
The farmer can short 3 contracts that have 3 months to maturity. If the price of cattle falls, the gain on the futures contract will offset the loss on the sale of the cattle. If the price of cattle rises, the gain on the sale of the cattle will be offset by the loss on the futures contract. Using futures contracts to hedge has the advantage that it can at no cost reduce risk to almost zero. Its disadvantage is that the farmer no longer gains from favorable movements in cattle prices.

### Problem 1.12.

The mining company can estimate its production on a month by month basis. It can then short futures contracts to lock in the price received for the gold. For example, if a total of 3,000 ounces are expected to be produced in September 2014 and October 2014, the price received for this production can be hedged by shorting a total of 30 October 2014 contracts.

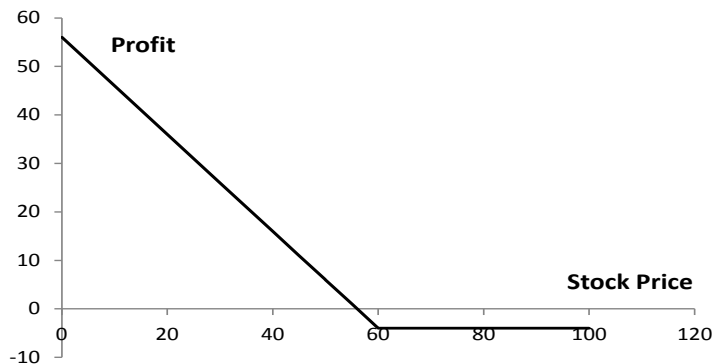
### Problem 1.13.

The holder of the option will gain if the price of the stock is above \$52.50 in March. (This ignores the time value of money.) The option will be exercised if the price of the stock is above \$50.00 in March. The profit as a function of the stock price is shown below.



### Problem 1.14.

The seller of the option will lose if the price of the stock is below \$56.00 in June. (This ignores the time value of money.) The option will be exercised if the price of the stock is below \$60.00 in June. The profit as a function of the stock price is shown below.



**Problem 1.20.**

- a) The trader sells 100 million yen for \$0.0080 per yen when the exchange rate is \$0.0074 per yen. The gain is  $100 \times 0.0006$  millions of dollars or \$60,000.
- b) The trader sells 100 million yen for \$0.0080 per yen when the exchange rate is \$0.0091 per yen. The loss is  $100 \times 0.0011$  millions of dollars or \$110,000.

**Problem 1.21.**

- a) The trader sells for 50 cents per pound something that is worth 48.20 cents per pound. Gain =  $(\$0.5000 - \$0.4820) \times 50,000 = \$900$ .
- b) The trader sells for 50 cents per pound something that is worth 51.30 cents per pound. Loss =  $(\$0.5130 - \$0.5000) \times 50,000 = \$650$ .

**Problem 2.11.**

There is a margin call if more than \$1,500 is lost on one contract. This happens if the futures price of frozen orange juice falls by more than 10 cents to below 150 cents per lb. \$2,000 can be withdrawn from the margin account if there is a gain on one contract of \$1,000. This will happen if the futures price rises by 6.67 cents to 166.67 cents per lb.

**Problem 2.15.**

The clearing house member is required to provide  $20 \times \$2,000 = \$40,000$  as initial margin for the new contracts. There is a gain of  $(50,200 - 50,000) \times 100 = \$20,000$  on the existing contracts. There is also a loss of  $(51,000 - 50,200) \times 20 = \$16,000$  on the new contracts. The member must therefore add

$$40,000 - 20,000 + 16,000 = \$36,000$$

to the margin account.

**Problem 2.16.**

Suppose  $F_1$  and  $F_2$  are the forward exchange rates for the contracts entered into July 1, 2013 and September 1, 2013, respectively. Suppose further that  $S$  is the spot rate on January 1, 2014. (All exchange rates are measured as yen per dollar). The payoff from the first contract is  $(S - F_1)$  million yen and the payoff from the second contract is  $(F_2 - S)$  million yen. The total payoff is therefore  $(S - F_1) + (F_2 - S) = (F_2 - F_1)$  million yen.

**Problem 2.23.**

The total profit is  $40,000 \times (0.9120 - 0.8830) = \$1,160$

If you are a hedger this is all taxed in 2014. If you are a speculator  $40,000 \times (0.9120 - 0.8880) = \$960$  is taxed in 2013 and  $40,000 \times (0.8880 - 0.8830) = \$200$  is taxed in 2014.

**Problem 3.12.**

Suppose that in Example 3.4 the company decides to use a hedge ratio of 0.8. How does the decision affect the way in which the hedge is implemented and the result?

If the hedge ratio is 0.8, the company takes a long position in 16 December oil futures contracts on June 8 when the futures price is \$8. It closes out its position on November 10. The spot price and futures price at this time are \$95 and \$92. The gain on the futures position is

$$(92 - 88) \times 16,000 = \$64,000$$

The effective cost of the oil is therefore

$$20,000 \times 95 - 64,000 = \$1,836,000$$

or \$91.80 per barrel. (This compares with \$91.00 per barrel when the company is fully hedged.)

**Problem 3.16.**

The optimal hedge ratio is

$$0.7 \times \frac{1.2}{1.4} = 0.6$$

The beef producer requires a long position in  $200,000 \times 0.6 = 120,000$  lbs of cattle. The beef producer should therefore take a long position in 3 December contracts closing out the position on November 15.

**Problem 3.18.**

A short position in

$$1.3 \times \frac{50,000 \times 30}{50 \times 1,500} = 26$$

contracts is required. It will be profitable if the stock outperforms the market in the sense that its return is greater than that predicted by the capital asset pricing model.

**Problem 4.10.**

The equivalent rate of interest with quarterly compounding is  $R$  where

$$e^{0.12} = \left(1 + \frac{R}{4}\right)^4$$

or

$$R = 4(e^{0.03} - 1) = 0.1218$$

The amount of interest paid each quarter is therefore:

$$10,000 \times \frac{0.1218}{4} = 304.55$$

or \$304.55.

**Problem 4.11.**

The bond pays \$2 in 6, 12, 18, and 24 months, and \$102 in 30 months. The cash price is

$$2e^{-0.04 \times 0.5} + 2e^{-0.042 \times 1.0} + 2e^{-0.044 \times 1.5} + 2e^{-0.046 \times 2} + 102e^{-0.048 \times 2.5} = 98.04$$

**Problem 4.14.**

The forward rates with continuous compounding are as follows: to

Year 2: 4.0%

Year 3: 5.1%

Year 4: 5.7%

Year 5: 5.7%

**Problem 5.9.**

*A one-year long forward contract on a non-dividend-paying stock is entered into when the stock price is \$40 and the risk-free rate of interest is 10% per annum with continuous compounding.*

- a) *What are the forward price and the initial value of the forward contract?*
- b) *Six months later, the price of the stock is \$45 and the risk-free interest rate is still 10%. What are the forward price and the value of the forward contract?*

- a) The forward price,  $F_0$ , is given by equation (5.1) as:

$$F_0 = 40e^{0.1 \times 1} = 44.21$$

or \$44.21. The initial value of the forward contract is zero.

- b) The delivery price  $K$  in the contract is \$44.21. The value of the contract,  $f$ , after six months is given by equation (5.5) as:

$$f = 45 - 44.21e^{-0.1 \times 0.5}$$

$$= 2.95$$

i.e., it is \$2.95. The forward price is:

$$45e^{0.1 \times 0.5} = 47.31$$

or \$47.31.

**Problem 5.10.**

*The risk-free rate of interest is 7% per annum with continuous compounding, and the dividend yield on a stock index is 3.2% per annum. The current value of the index is 150. What is the six-month futures price?*

Using equation (5.3) the six month futures price is

$$150e^{(0.07 - 0.032) \times 0.5} = 152.88$$

or \$152.88.

**Problem 5.14.**

The theoretical futures price is

$$0.8000e^{(0.05-0.02)\times 2/12} = 0.8040$$

The actual futures price is too high. This suggests that an arbitrageur should buy Swiss francs and short Swiss francs futures.

**Problem 5.15.**

The present value of the storage costs for nine months are

$$0.12 + 0.12e^{-0.10\times 0.25} + 0.12e^{-0.10\times 0.5} = 0.351$$

or \$0.351. The futures price is from equation (5.11) given by  $F_0$  where

$$F_0 = (30 + 0.351)e^{0.1\times 0.75} = 32.72$$

i.e., it is \$32.72 per ounce.

**Problem 6.8.**

The cash price of the Treasury bill is

$$100 - \frac{90}{360} \times 10 = \$97.50$$

The annualized continuously compounded return is

$$\frac{365}{90} \ln\left(1 + \frac{2.5}{97.5}\right) = 10.27\%$$

**Problem 6.9.**

The number of days between January 27, 2013 and May 5, 2013 is 98. The number of days between January 27, 2013 and July 27, 2013 is 181. The accrued interest is therefore

$$6 \times \frac{98}{181} = 3.2486$$

The quoted price is 110.5312. The cash price is therefore

$$110.5312 + 3.2486 = 113.7798$$

or \$113.78.

**Problem 6.10.**

The cheapest-to-deliver bond is the one for which

Quoted Price – Futures Price  $\times$  Conversion Factor is least. Calculating this factor for each of the 4 bonds we get

$$\text{Bond 1: } 125.15625 - 101.375 \times 1.2131 = 2.178$$

$$\text{Bond 2: } 142.46875 - 101.375 \times 1.3792 = 2.652$$

$$\text{Bond 3: } 115.96875 - 101.375 \times 1.1149 = 2.946$$

$$\text{Bond 4: } 144.06250 - 101.375 \times 1.4026 = 1.874$$

Bond 4 is therefore the cheapest to deliver.

**Problem 6.11.**

There are 176 days between February 4 and July 30 and 181 days between February 4 and August 4. The cash price of the bond is, therefore:

$$110 + \frac{176}{181} \times 6.5 = 116.32$$

The rate of interest with continuous compounding is  $2 \ln 1.06 = 0.1165$  or 11.65% per annum. A coupon of 6.5 will be received in 5 days (= 0.01370 years) time. The present value of the coupon is

$$6.5e^{-0.01370 \times 0.1165} = 6.49$$

The futures contract lasts for 62 days (= 0.1699 years). The cash futures price if the contract were written on the 13% bond would be

$$(116.32 - 6.49)e^{0.1699 \times 0.1165} = 112.03$$

At delivery there are 57 days of accrued interest. The quoted futures price if the contract were written on the 13% bond would therefore be

$$112.03 - 6.5 \times \frac{57}{184} = 110.01$$

Taking the conversion factor into account the quoted futures price should be:

$$\frac{110.01}{1.5} = 73.34$$

**Problem 7.9.**

The spread between the interest rates offered to X and Y is 0.8% per annum on fixed rate investments and 0.0% per annum on floating rate investments. This means that the total apparent benefit to all parties from the swap is 0.8% per annum. Of this 0.2% per annum will go to the bank. This leaves 0.3% per annum for each of X and Y. In other words, company X should be able to get a fixed-rate return of 8.3% per annum while company Y should be able to get a floating-rate return LIBOR + 0.3% per annum. The required swap is shown below. The bank earns 0.2%, company X earns 8.3%, and company Y earns LIBOR + 0.3%.



**Problem 7.10.**

At the end of year 3 the financial institution was due to receive \$500,000 (= 0.5 × 10% of \$10 million) and pay \$450,000 (= 0.5 × 9% of \$10 million). The immediate loss is therefore \$50,000. To value the remaining swap we assume that forward rates are realized. All forward rates are 8% per annum. The remaining cash flows are therefore valued on the assumption that the floating payment is  $0.5 \times 0.08 \times 10,000,000 = \$400,000$  and the net payment that would be received is  $500,000 - 400,000 = \$100,000$ . The total cost of default is therefore the cost of foregoing the following cash flows:

3 year: \$50,000  
 3.5 year: \$100,000  
 4 year: \$100,000  
 4.5 year: \$100,000  
 5 year: \$100,000

Discounting these cash flows to year 3 at 4% per six months, we obtain the cost of the default as \$413,000.

**Problem 7.11.**

When interest rates are compounded annually

$$F_0 = S_0 \left( \frac{1+r}{1+r_f} \right)^T$$

where  $F_0$  is the  $T$ -year forward rate,  $S_0$  is the spot rate,  $r$  is the domestic risk-free rate, and  $r_f$  is the foreign risk-free rate. As  $r = 0.08$  and  $r_f = 0.03$ , the spot and forward exchange rates at the end of year 6 are

Spot: 0.8000  
 1 year forward: 0.8388  
 2 year forward: 0.8796  
 3 year forward: 0.9223  
 4 year forward: 0.9670

The value of the swap at the time of the default can be calculated on the assumption that forward rates are realized. The cash flows lost as a result of the default are therefore as follows:

<i>Year</i>	<i>Dollar Paid</i>	<i>CHF Received</i>	<i>Forward Rate</i>	<i>Dollar Equiv of CHF Received</i>	<i>Cash Flow Lost</i>
6	560,000	300,000	0.8000	240,000	-320,000
7	560,000	300,000	0.8388	251,600	-308,400
8	560,000	300,000	0.8796	263,900	-296,100
9	560,000	300,000	0.9223	276,700	-283,300
10	7,560,000	10,300,000	0.9670	9,960,100	2,400,100

Discounting the numbers in the final column to the end of year 6 at 8% per annum, the cost of the default is \$679,800.

Note that, if this were the only contract entered into by company Y, it would make no sense for the company to default just before the exchange of payments at the end of year 6 as the exchange has a positive value to company Y. In practice, company Y is likely to be defaulting and declaring bankruptcy for reasons unrelated to this particular transaction.

**Problem 8.9.**

Investors underestimated how high the default correlations between mortgages would be in stressed market conditions. Investors also did not always realize that the tranches underlying ABS CDOs were usually quite thin so that they were either totally wiped out or untouched. There was an unfortunate tendency to assume that a tranche with a particular rating could be considered to be the same as a bond with that rating. This assumption is not valid for the reasons just mentioned.

**Problem 8.10.**

“Agency costs” is a term used to describe the costs in a situation where the interests of two parties are not perfectly aligned. There were potential agency costs between a) the originators of mortgages and investors and b) employees of banks who earned bonuses and the banks themselves.

**Problem 8.11.**

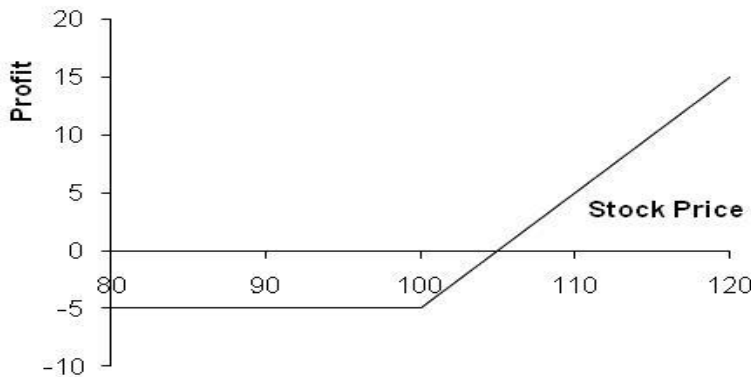
Typically an ABS CDO is created from the BBB-rated tranches of an ABS. This is because it is difficult to find investors in a direct way for the BBB-rated tranches of an ABS.

**Problem 8.12.**

As default correlation increases, the senior tranche of a CDO becomes more risky because it is more likely to suffer losses. As default correlation increases, the equity tranche becomes less risky. To understand why this is so, note that in the limit when there is perfect correlation there is a high probability that there will be no defaults and the equity tranche will suffer no losses.

**Problem 9.9.**

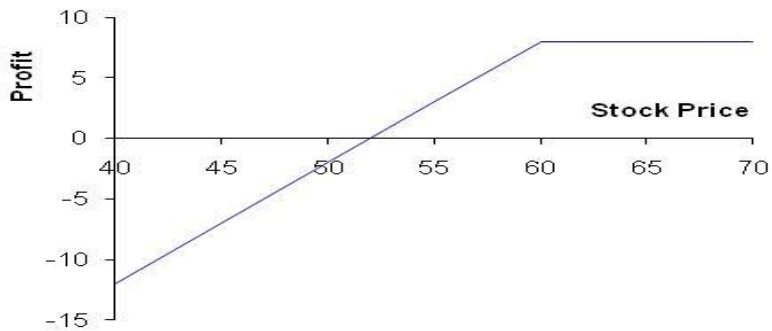
Ignoring the time value of money, the holder of the option will make a profit if the stock price at maturity of the option is greater than \$105. This is because the payoff to the holder of the option is, in these circumstances, greater than the \$5 paid for the option. The option will be exercised if the stock price at maturity is greater than \$100. Note that if the stock price is between \$100 and \$105 the option is exercised, but the holder of the option takes a loss overall. The profit from a long position is as shown below.





**Problem 9.10.**

Ignoring the time value of money, the seller of the option will make a profit if the stock price at maturity is greater than \$52.00. This is because the cost to the seller of the option is in these circumstances less than the price received for the option. The option will be exercised if the stock price at maturity is less than \$60.00. Note that if the stock price is between \$52.00 and \$60.00 the seller of the option makes a profit even though the option is exercised. The profit from the short position is as shown below.

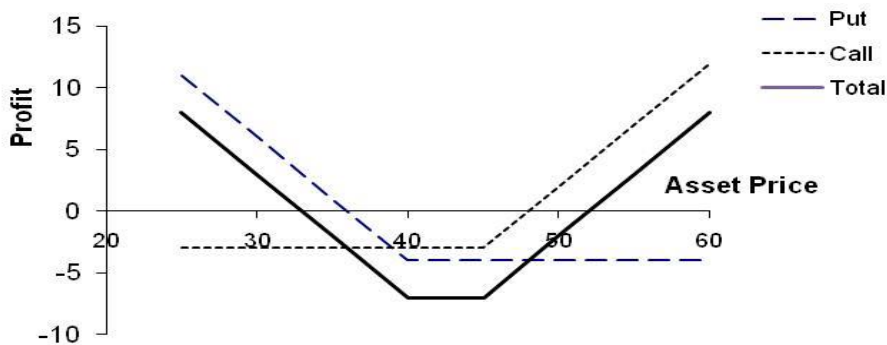


**Problem 9.12.**

The following figure shows the variation of the trader’s position with the asset price. We can divide the alternative asset prices into three ranges:

- a) When the asset price less than \$40, the put option provides a payoff of  $40 - S_T$  and the call option provides no payoff. The options cost \$7 and so the total profit is  $33 - S_T$ .
- b) When the asset price is between \$40 and \$45, neither option provides a payoff. There is a net loss of \$7.
- c) When the asset price greater than \$45, the call option provides a payoff of  $S_T - 45$  and the put option provides no payoff. Taking into account the \$7 cost of the options, the total profit is  $S_T - 52$ .

The trader makes a profit (ignoring the time value of money) if the stock price is less than \$33 or greater than \$52. This type of trading strategy is known as a strangle and is discussed in Chapter 11.



**Problem 10.9.**

The lower bound is

$$80 - 75e^{-0.1 \times 0.5} = \$8.66$$

**Problem 10.10**

The lower bound is

$$65e^{-0.05 \times 2/12} - 58 = \$6.46$$

**Problem 10.11.**

The present value of the strike price is  $60e^{-0.12 \times 4/12} = \$57.65$ . The present value of the dividend is  $0.80e^{-0.12 \times 1/12} = 0.79$ . Because

$$5 < 64 - 57.65 - 0.79$$

the condition in equation (10.8) is violated. An arbitrageur should buy the option and short the stock. This generates  $64 - 5 = \$59$ . The arbitrageur invests \$0.79 of this at 12% for one month to pay the dividend of \$0.80 in one month. The remaining \$58.21 is invested for four months at 12%. Regardless of what happens a profit will materialize.

If the stock price declines below \$60 in four months, the arbitrageur loses the \$5 spent on the option but gains on the short position. The arbitrageur shorts when the stock price is \$64, has to pay dividends with a present value of \$0.79, and closes out the short position when the stock price is \$60 or less. Because \$57.65 is the present value of \$60, the short position generates at least  $64 - 57.65 - 0.79 = \$5.56$  in present value terms. The present value of the arbitrageur's gain is therefore at least  $5.56 - 5.00 = \$0.56$ .

If the stock price is above \$60 at the expiration of the option, the option is exercised. The arbitrageur buys the stock for \$60 in four months and closes out the short position. The present value of the \$60 paid for the stock is \$57.65 and as before the dividend has a present value of \$0.79. The gain from the short position and the exercise of the option is therefore exactly  $64 - 57.65 - 0.79 = \$5.56$ . The arbitrageur's gain in present value terms is  $5.56 - 5.00 = \$0.56$ .

**Problem 10.12.**

In this case the present value of the strike price is  $50e^{-0.06 \times 1/12} = 49.75$ . Because

$$2.5 < 49.75 - 47.00$$

the condition in equation (10.5) is violated. An arbitrageur should borrow \$49.50 at 6% for one month, buy the stock, and buy the put option. This generates a profit in all circumstances.

If the stock price is above \$50 in one month, the option expires worthless, but the stock can be sold for at least \$50. A sum of \$50 received in one month has a present value of \$49.75 today.

The strategy therefore generates profit with a present value of at least \$0.25.

If the stock price is below \$50 in one month the put option is exercised and the stock owned is sold for exactly \$50 (or \$49.75 in present value terms). The trading strategy therefore generates a profit of exactly \$0.25 in present value terms.

**Problem 11.10.**

A bull spread is created by buying the \$30 put and selling the \$35 put. This strategy gives rise to an initial cash inflow of \$3. The outcome is as follows:

Stock Price	Payoff	Profit
$S_T \geq 35$	0	3
$30 \leq S_T < 35$	$S_T - 35$	$S_T - 32$
$S_T < 30$	-5	-2

A bear spread is created by selling the \$30 put and buying the \$35 put. This strategy costs \$3 initially. The outcome is as follows

Stock Price	Payoff	Profit
$S_T \geq 35$	0	-3
$30 \leq S_T < 35$	$35 - S_T$	$32 - S_T$
$S_T < 30$	5	2

**Problem 11.11.**

Define  $c_1$ ,  $c_2$ , and  $c_3$  as the prices of calls with strike prices  $K_1$ ,  $K_2$  and  $K_3$ . Define  $p_1$ ,  $p_2$  and  $p_3$  as the prices of puts with strike prices  $K_1$ ,  $K_2$  and  $K_3$ . With the usual notation

$$c_1 + K_1 e^{-rT} = p_1 + S$$

$$c_2 + K_2 e^{-rT} = p_2 + S$$

$$c_3 + K_3 e^{-rT} = p_3 + S$$

Hence

$$c_1 + c_3 - 2c_2 + (K_1 + K_3 - 2K_2)e^{-rT} = p_1 + p_3 - 2p_2$$

Because  $K_2 - K_1 = K_3 - K_2$ , it follows that  $K_1 + K_3 - 2K_2 = 0$  and

$$c_1 + c_3 - 2c_2 = p_1 + p_3 - 2p_2$$

The cost of a butterfly spread created using European calls is therefore exactly the same as the cost of a butterfly spread created using European puts.

**Problem 11.12.**

A straddle is created by buying both the call and the put. This strategy costs \$10. The profit/loss is shown in the following table:

Stock Price	Payoff	Profit
$S_T > 60$	$S_T - 60$	$S_T - 70$
$S_T \leq 60$	$60 - S_T$	$50 - S_T$

This shows that the straddle will lead to a loss if the final stock price is between \$50 and \$70.

**Problem 12.9.**

At the end of two months the value of the option will be either \$4 (if the stock price is \$53) or \$0 (if the stock price is \$48). Consider a portfolio consisting of:

+ $\Delta$  : shares

-1 : option

The value of the portfolio is either  $48\Delta$  or  $53\Delta - 4$  in two months. If

$$48\Delta = 53\Delta - 4$$

i.e.,

$$\Delta = 0.8$$

the value of the portfolio is certain to be 38.4. For this value of  $\Delta$  the portfolio is therefore riskless. The current value of the portfolio is:

$$0.8 \times 50 - f$$

where  $f$  is the value of the option. Since the portfolio must earn the risk-free rate of interest

$$(0.8 \times 50 - f)e^{0.10 \times 2/12} = 38.4$$

i.e.,

$$f = 2.23$$

The value of the option is therefore \$2.23.

This can also be calculated directly from equations (12.2) and (12.3).  $u = 1.06$ ,  $d = 0.96$  so that

$$p = \frac{e^{0.10 \times 2/12} - 0.96}{1.06 - 0.96} = 0.5681$$

and

$$f = e^{-0.10 \times 2/12} \times 0.5681 \times 4 = 2.23$$

**Problem 12.10.**

At the end of four months the value of the option will be either \$5 (if the stock price is \$75) or \$0 (if the stock price is \$85). Consider a portfolio consisting of:

- $\Delta$  : shares

+1 : option

(Note: The delta,  $\Delta$  of a put option is negative. We have constructed the portfolio so that it is +1 option and  $-\Delta$  shares rather than -1 option and  $+\Delta$  shares so that the initial investment is positive.)

The value of the portfolio is either  $-85\Delta$  or  $-75\Delta + 5$  in four months. If

$$-85\Delta = -75\Delta + 5$$

i.e.,

$$\Delta = -0.5$$

the value of the portfolio is certain to be 42.5. For this value of  $\Delta$  the portfolio is therefore riskless. The current value of the portfolio is:

$$0.5 \times 80 + f$$

where  $f$  is the value of the option. Since the portfolio is riskless

$$(0.5 \times 80 + f)e^{0.05 \times 4/12} = 42.5$$

i.e.,

$$f = 1.80$$

The value of the option is therefore \$1.80.

This can also be calculated directly from equations (12.2) and (12.3).  $u = 1.0625$ ,  $d = 0.9375$  so that

$$p = \frac{e^{0.05 \times 4/12} - 0.9375}{1.0625 - 0.9375} = 0.6345$$

$1 - p = 0.3655$  and

$$f = e^{-0.05 \times 4/12} \times 0.3655 \times 5 = 1.80$$

**Problem 12.11.**

At the end of three months the value of the option is either \$5 (if the stock price is \$35) or \$0 (if the stock price is \$45).

Consider a portfolio consisting of:

$-\Delta$  : shares

+1 : option

(Note: The delta,  $\Delta$ , of a put option is negative. We have constructed the portfolio so that it is +1 option and  $-\Delta$  shares rather than  $-1$  option and  $+\Delta$  shares so that the initial investment is positive.)

The value of the portfolio is either  $-35\Delta + 5$  or  $-45\Delta$ . If:

$$-35\Delta + 5 = -45\Delta$$

i.e.,

$$\Delta = -0.5$$

the value of the portfolio is certain to be 22.5. For this value of  $\Delta$  the portfolio is therefore riskless. The current value of the portfolio is

$$-40\Delta + f$$

where  $f$  is the value of the option. Since the portfolio must earn the risk-free rate of interest

$$(40 \times 0.5 + f) \times 1.02 = 22.5$$

Hence

$$f = 2.06$$

i.e., the value of the option is \$2.06.

This can also be calculated using risk-neutral valuation. Suppose that  $p$  is the probability of an upward stock price movement in a risk-neutral world. We must have

$$45p + 35(1 - p) = 40 \times 1.02$$

i.e.,

$$10p = 5.8$$

or:

$$p = 0.58$$

The expected value of the option in a risk-neutral world is:

$$0 \times 0.58 + 5 \times 0.42 = 2.10$$

This has a present value of

$$\frac{2.10}{1.02} = 2.06$$

This is consistent with the no-arbitrage answer.

**Problem 12.12.**

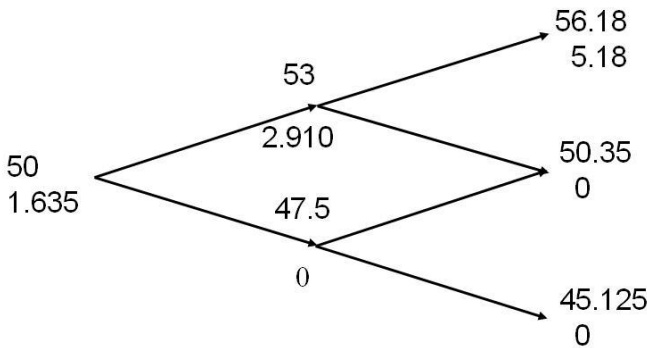
A tree describing the behavior of the stock price is shown below. The risk-neutral probability of an up move,  $p$ , is given by

$$p = \frac{e^{0.05 \times 3/12} - 0.95}{1.06 - 0.95} = 0.5689$$

There is a payoff from the option of  $56.18 - 51 = 5.18$  for the highest final node (which corresponds to two up moves) zero in all other cases. The value of the option is therefore

$$5.18 \times 0.5689^2 \times e^{-0.05 \times 6/12} = 1.635$$

This can also be calculated by working back through the tree as indicated in Figure S12.1. The value of the call option is the lower number at each node in the figure.



**Problem 13.9.**

- a) The required probability is the probability of the stock price being above \$40 in six months time. Suppose that the stock price in six months is  $S_T$ . The probability distribution of  $\ln S_T$  is

$$\phi \left\{ \ln 38 + \left( 0.16 - \frac{0.35^2}{2} \right) 0.5, 0.35^2 \times 0.5 \right\}$$

i.e.,

$$\phi(3.687, 0.247^2)$$

Since  $\ln 40 = 3.689$ , the required probability is

$$1 - N \left( \frac{3.689 - 3.687}{0.247} \right) = 1 - N(0.008)$$

From normal distribution tables  $N(0.008) = 0.5032$  so that the required probability is 0.4968.

- b) In this case the required probability is the probability of the stock price being less than \$40 in six months. It is

$$1 - 0.4968 = 0.5032$$

**Problem 13.14.**

In this case,  $S_0 = 69$ ,  $K = 70$ ,  $r = 0.05$ ,  $\sigma = 0.35$ , and  $T = 0.5$ .

$$d_1 = \frac{\ln(69/70) + (0.05 + 0.35^2/2) \times 0.5}{0.35\sqrt{0.5}} = 0.1666$$

$$d_2 = d_1 - 0.35\sqrt{0.5} = -0.0809$$

The price of the European put is

$$70e^{-0.05 \times 0.5} N(0.0809) - 69N(-0.1666)$$

$$= 70e^{-0.025} \times 0.5323 - 69 \times 0.4338$$

$$= 6.40$$

or \$6.40.