Mathematics for Elementary School: Collaboration Between Mathematics and Elementary Education

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Two Essential Components for Training Successful Teachers of Mathematics in Elementary School

- A) Mastery of Content Knowledge
- B) Skill at communicating correctly and effectively in the classroom

Integration of Both Components in Teaching Content Courses

- Student discussion/explanation in mathematically rich contexts (exploring definitions of even numbers, using arithmetic algorithms smartly,,,)
- Conceptual and procedural understanding (Ma Q1: 52 – 27 = 35)
- Using multiple representations & making connections between models and understanding (chip model for subtraction)
- Solving demanding problems (strip diagrams for understanding algebra problems)

CSUN Teacher Education for Elementary Mathematics Teaching

Math 210:
Number and Operations Class

3 semester units

Math 310:
Geometry, Probability, Statistics Class

3 units +

Math 310 Lab:
Investigate Math Concepts through Manipulatives

1 unit

Mathematics Methods Class

2 units

1st Student Teaching and Seminar

4 units

2nd Student Teaching and Seminar

7 units

Engagement of Math Faculty

- Texts
- Common final (as a stimulus to interested discussion by instructors)
- MKT surveys (sample questions, results in content & in ed courses and using questions to explore how we teach)
- Videos of our teaching

CSUN Teacher Education: Content Courses

Math 210:
Number and
Operations Class

Texts Used: Billstein, Sowder, Parker & Baldridge

Now: Parker & Baldridge, Beckmann

Math 310:
Geometry Class

Text Used: Billstein, Sowder

Now: Billstein, Beckmann

PRIMARY ASSESSMENT TOOL

Using Ball &Hill (2004) survey of MKT: Mathematical Knowledge for Teaching (LMT/CKTM)

Advantages:

1. already developed
2. scaled (in-service teachers, z-scores)
3. reliable
4. valid
5. correlated with in-service teachers' higher pupil gains

Drawbacks:

1. Time & convenience (now online?)
2. Not all math sections participate…yet.

Sample Question:
Number & Operations Content Knowledge

http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf

- Mrs. Harris was working on divisibility rules. She told her class that a number is divisible by 4 if the last two digits are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4? (Mark ONE answer.)

a) Four is an even number, and odd numbers are not divisible by even numbers,

b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.)

c) Every other even number is divisible by 4, for example, 24 and 28 but not 26

d) It only works when the sum of the last two digits is an even number.

Mathematical Knowledge for Teaching Reporting Domains

- Number & Operations (Form A, B, C)
  - CK = Content Knowledge
  - KSC = Knowledge of students & content
  - PFA = Patterns, functions & algebra
- Geometry (Form A and B)
  - CK = Content Knowledge

Research Plan

Math 210:
Number and
Operations Class

MKT Number & Operations
Pre/Post

Math 310:
Geometry, Probability,
Statistics Class

MKT - Geometry
Pre/Post

Mathematics Methods
Class

MKT - Number & Operations CK & KSC Post

1st Student Teaching
and Seminar

2nd Student Teaching
and Seminar

MKT - Number & Operations KSC (CK)
MKT - Geometry
Supervisor Lesson Observations/ Lesson Evaluations/
Assessment Project

Mathematical Knowledge for Teaching

Results for Geometry

Math 310 Pre-Test and Post-Test in *z-scores*

	Spr 05	Fall 05	Spr 06	Fall 06	Spr 07
Pre test	**-0.5**	**-0.57**	**-0.66**	**-0.36**	**-0.49**
Post Test	**0.14**	**0.03**	**0.14**	**-0.19**	**-0.01**
Gain	**0.66**	**0.6**	**0.77**	**0.19**	**0.44**
N	80	75	123	58	85

Tentative Interpretation of this Data

- The positive gain scores indicate that Math 310 students increase their Mathematical Knowledge for Teaching as a result of taking this course.

Math Knowledge for Teaching Results: Numbers & Operations

CK = Content Knowledge

	Spr 05	Fall 05	Spr 06	Fall 06	Spr 07
Pre test	**-0.81**	**-0.72**	**-0.81**	**-0.69**	**-0.66**
Post Test	**-0.42**	**-0.35**	**-0.27**	**-0.39**	**-0.54**
Gain	**0.49**	**0.4**	**0.37**	**0.17**	**0.12**
N	26	41	29	77	61

KSC = Knowledge of Students & Content

	Spr 05	Fall 05	Spr 06	Fall 06	Spr 07
Pre test	**-0.4**	**-0.57**	**-0.44**	**-0.74**	**-0.55**
Post Test	**-0.73**	**-0.5**	**-0.61**	**-0.44**	**-0.39**
Gain	**-0.35**	**0.11**	**-0.16**	**0.13**	**0.15**
N	26	41	29	77	61

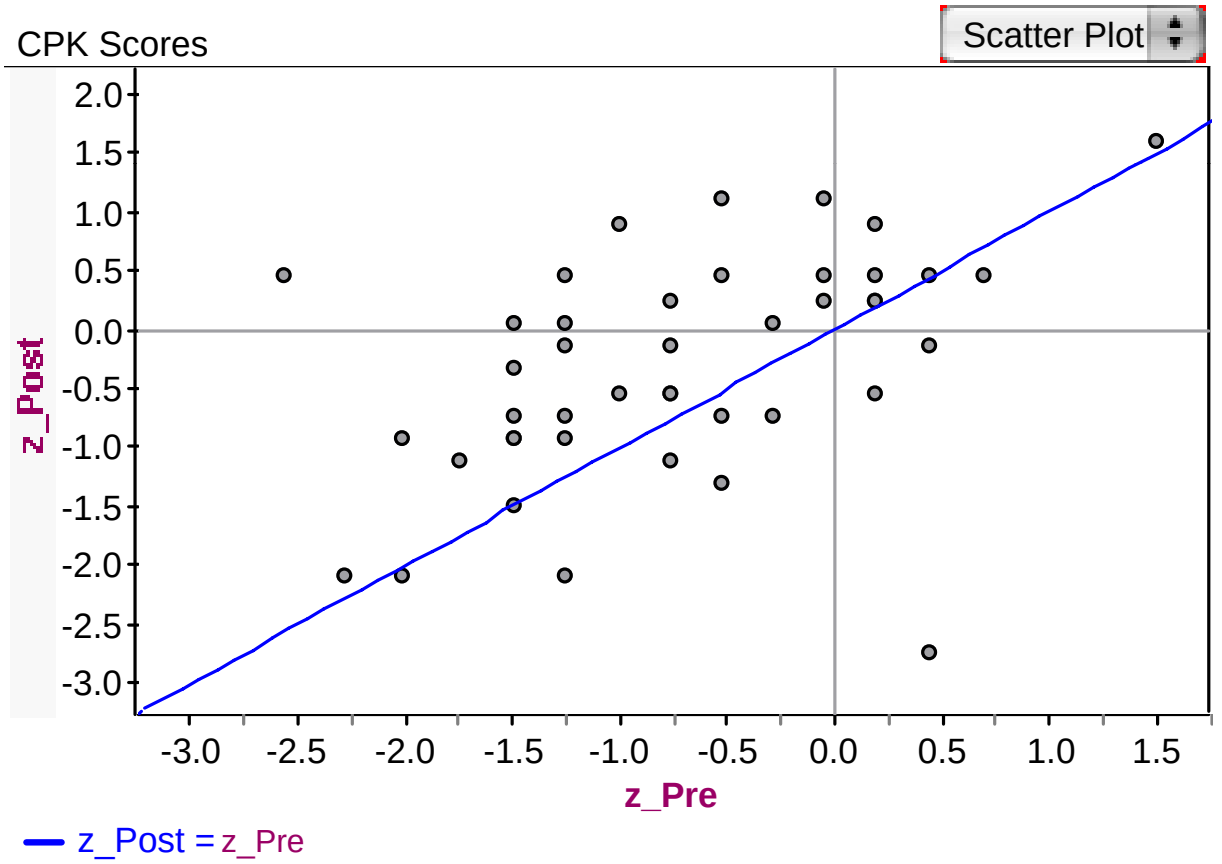
PFA = Patterns, Functions & Algebra

	Spr 05	Fall 05	Spr 06	Fall 06	Spr 07
Pre test		**0.22**	**-0.62**	**-0.57**	**-0.54**
Post Test		**-0.74**	**-0.65**	**-0.03**	**0**
Gain		**-0.95**	**0.05**	**0.46**	**0.51**
N		41	29	77	61

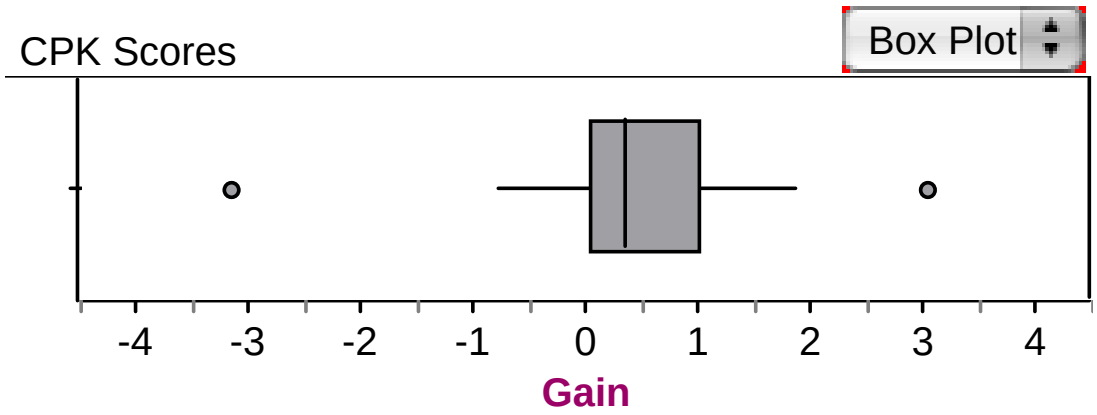
N = number taking both pre and post tests, while the pre test averages include all those taking the test. Similarly for post test averages.

Plot Showing Results of Number and Operations Content Knowledge in Fall 2005

Post-Test *z*-scores plotted against Pre-Test z-scores showing improvement



Gains on Number and Operations Content Knowledge Construct in Fall 2005



Statistical Significance
Fall 2005 Number and Operations Content Knowledge Construct

Test of Only Took Both

Test Mean

Attribute (numeric): Gain

Ho: population mean of **Gain** equals **0**
Ha: population mean of **Gain** **is not equal to 0**

Count:	**43**
Mean:	**0.44007**
Std dev:	**0.959509**
Std error:	**0.146324**
Student's t:	**3.008**
DF:	**42**
P-value:	**0.0044**

Conclusions & Questions: Math Course Data

- Students show significant gains in Number and Operations Content Knowledge.
  - We should continue & extend current practices.
  - Does their learning endure?
  - Do the education courses need to devote a major part of their time to reinforcing students knowledge of arithmetic content?
- Change?In numbers & operations KSC (knowledge of students and content) & PFA (patterns, functions and algebra) there is not consistent improvement. Not unexpected since this is not the focus of the courses. We continue to collect data for gauging progress later in the program after math methods and field experience which do focus on these goals.

MKT Data From Education Courses

Mathematics Methods Class

1st Student Teaching and Seminar

2nd Student Teaching and Seminar

What are Teacher Candidates Learning?
Is this knowledge carried over to Student Teaching?

Table 1: Means (z-scores) of MKT Geometry Measures for Three Groups of Teacher Candidates (Longitudinal Study)

MKT Geometry Measures	4-Year Undergraduate Cohort n = 25	2-Year Undergraduate Cohort n =17	2-Year Undergraduate Control Group n = 33
Pre Test Geometry Class	- 0.17	- 0.45	---
Post Test Geometry Class	0.85	0.10	---
Student Teaching Post Test	0.58	- 0.20	- 0.41
Gains	.75	.25	--

Table 2: Means (z-scores) of MKT Number and Operations Measures - Knowledge of Students and Content - for Three Groups of Teacher Candidates

MKT Number & Operations- **Knowledge of Students and Content** Measures	4-Year Undergraduate Cohort *n* = 25	2-Year Undergraduate Cohort *n = 17*	2-Year Undergraduate Control Group *n* = 33
Mathematics Methods Class Post-test	- 0.004	- 0.481	-0.236
Student Teaching Post-test	0.270	- 0.067	- 0.089
Gains	0.274	0.414	0.147

Results for All Undergraduate Teacher Candidates

Table 3. Mean z-scores on Number and Operations Measures for all Undergraduate Teacher Candidates from Spring 2005 to Fall 2007

Classes	n	Number & Operations: **Content Knowledge**	Number & Operations: **Knowledge of Students and Content**	Number & Operations: **Patterns, Functions, Algebra**
Pre Test Math 210	234	-0.72	-0.59	-0.41
Post Test Math 210	234	-0.41	-0.49	-0.24
Post Test Math Methods	225	-0.09	-0.21	1.12
Post Test 2nd Sem. St. Teaching	75	--	-0.009	--

Results for Undergraduate and 5th year Program on Number and Operations Measures

Table 4. Mean z-scores on Number and Operations Measures on Mathematics Methods Post Tests for Two CSUN Programs

Program	n	Number & Operations: **Content Knowledge**	Number & Operations: **Knowledge of Students and Content**	Number & Operations: **Patterns, Functions, Algebra**
UNDER-GRADUATE Program	225	-0.09	-0.21	1.13
5th Year Program	110	-0.17	-0.21	1.04

Questions To Explore Using Evidence

- Are we improving our students' MKT CK (content knowledge) in math content courses?
- Are we improving our students' MKT KSC in Math methods courses?
- Does success in MKT CK in content courses (post test scores or gains) lead to improved scores in other domains in Math Methods courses and in student teaching?
- Is it important to re-emphasize MKT CK in math methods courses? i.e. does more time spent on this improve candidates later success in student teaching or MKT KSC scores.?
- Do high gains or high post test scores correlate to any background factors (e.g. ELM scores, HS gpa, CSUN gpa, ELL status, SAT, % of ELL's in their HS, ….)?

Questions To Explore Using Evidence

- Do high gain scores (vs. high post test scores) correlate with success in leading their students to higher gains on achievement tests?
- Meta question: Do we care more about post test scores or gains? Look at successful teachers to see.—DB's results are for high scores, not gains.
- Are some groups more likely to have higher gains ("smarter" = better able to learn?)
- Why do we have pre-service post test scores in Geometry CK higher than 0 (representing the average for in-service teachers)? (Geom is not tested on state exams?, in-service teachers don't use geom and so forget geom.?...)

Long Term Considerations:

- What is the effect of 1 or 2 math content courses on students several years later?
- “Do our graduates teach the way we think we are training them to?”—HOW DO WE IMPLEMENT THIS?