1. A beginning library of elementary functions

2. Graphs of elementary functions

3. Shifts and stretches

4. Piecewise -defined functions

**Beginning Library**

- identity function $f(x) = x$
- absolute value function $f(x) = |x|$
- square function $f(x) = x^2$
- square-root function $f(x) = \sqrt{x}$
- piecewise defined functions
Identity and Absolute value functions

Identity function

Expression: \( f(x) = x \)

Domain: all numbers \((-\infty, \infty)\)

Range: all numbers \((-\infty, \infty)\)

Absolute-value function

Expression: \( f(x) = |x| \) or \( \text{abs} (x) \)

Domain: all numbers \((-\infty, \infty)\)

Range: \( x \geq 0, [0, \infty) \)

Square and Square-root functions

Square function

Expression: \( f(x) = x^2 \)

Domain: all numbers \((-\infty, \infty)\)

Range: \( x \geq 0, [0, \infty) \)

Square-root function

Expression: \( f(x) = \sqrt{x} \)

Domain: \( x \geq 0, [0, \infty) \)

Range: \( x \geq 0, [0, \infty) \)
Transformations

• vertical translations (shift)
  Business shifting up: Suppose $x$ is the number of items you produce and $C(x)$ is the cost to produce $x$ items. If your fixed costs (e.g. rent) increases by $5$, then the cost curve will shift up 5 units.

• vertical stretch
  Business stretching up: Suppose $x$ is the number of items you produce and $C(x) = 10x$ is the cost to produce $x$ items. If your variable costs to produce items (e.g. you are taxed on each unit produced) increases by $2$, then the cost curve will be stretched up by a factor of 2.

• horizontal translation (shift)
  Business shifting left: Suppose the units on the $x$-axis are years starting in 2000 (i.e. $x = 0$ is the year 2000) and $P(x)$ is your profits for the year. In updating your graphs for your manager you want to have the graph "start" with the year 2002. Then you want to ...

• reflections (lab sessions)

Vertical shift

Vertical shift 5 units up

The graph of $f(x)$ is blue (dark line).

The graph of $f(x) + 5$ is red (light line).

The vertical distance between the curves is 5.
**Vertical stretch**

Vertical stretch by a factor of 2:

![Graph of vertical stretch](image)

The graph of $f(x)$ is blue (dark line).

The graph of $2f(x)$ is red (light line).

The vertical distance from the $x$-axis of the graph of $2f(x)$ is twice that of $f(x)$.

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**Horizontal shift**

Horizontal shift two units to the right

![Graph of horizontal shift](image)

The graph of $f(x)$ is blue (dark).

The graph of $f(x - 2)$ is red (light).

The horizontal distance between the curves is 2.
Horizontal shift

Horizontal shift two units to the left

The graph of $h(x)$ is red (light).

The graph of $h(x + 2)$ is blue (dark).

The horizontal distance between the curves is 2.

Practice: Graph these functions

$y = 2|x|$
Practice: Each function corresponds to geometric description

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x - 5)$</td>
<td>horizontal shift 5 units to the right</td>
</tr>
<tr>
<td>$f(x) + 7$</td>
<td></td>
</tr>
<tr>
<td>$3f(x)$</td>
<td></td>
</tr>
<tr>
<td>$f(x - 3) - 1$</td>
<td>vertical shift 2 units up</td>
</tr>
<tr>
<td></td>
<td>vertical shrink by a factor of $1/2$</td>
</tr>
<tr>
<td></td>
<td>horizontal shift 4 units to left</td>
</tr>
</tbody>
</table>
Piecewise defined functions, an example

A car rental agency charges $30 per day (or partial day) or $150 per week, whichever is least. What is the rental cost $C(x)$ for $x$ days?

Fill in the charges for the values of $x$:

\[
\begin{array}{c|cccccccccc}
  x & 1.0 & 2.0 & 2.6 & 3.0 & 3.1 & 4.0 & 4.2 & 5.0 & 6.0 & 7.0 & 7.1 \\
  \hline
  C(x) & 30 & 60 & 90 & 90 & 120 & 120 & 150 & 150 & 150 & 150 & 180 \\
\end{array}
\]

Example from business continued:

A car rental agency charges $30 per day (or partial day) or $150 per week, whichever is least. What is the rental cost $C(x)$ for $x$ days?

Fill in the charges for the values of $x$:
Example from business

A car rental agency charges $30 per day (or partial day) or $150 per week, whichever is least. Graph the cost function $C(x)$.

Example from business $T(x)$ is the tax on taxable income of $x$.

The federal income tax rate is

<table>
<thead>
<tr>
<th>Between</th>
<th>But Not Over</th>
<th>Base Tax</th>
<th>Rate</th>
<th>Of the Amount Over</th>
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<tbody>
<tr>
<td>$0</td>
<td>$7,550</td>
<td>0</td>
<td>10%</td>
<td>$0.00</td>
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<tr>
<td>$7,550</td>
<td>$30,650</td>
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<td>15%</td>
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<td>$15,107.50</td>
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<td>$336,550</td>
<td>$37,675.50</td>
<td>33%</td>
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<td>$336,550</td>
<td>$97,653.00</td>
<td>35%</td>
<td>$336,550</td>
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If you have a taxable income of $x = 110,000$, your tax is

\[
T(110,000) = \text{Base Tax} + (\text{Rate} \times \text{Amount Over}) \\
= 15,107.50 + [.28 \times (110,000 - 74,200)] \\
= 15,107.50 + [.28 \times 35,800] \\
= 15,107.50 + 10,024.00 \\
= 25,131.50
\]

The graph of \(T(x)\):

![Graph of T(x)](image-url)
The equations for $T(x)$:

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For income between $74,200$ and $154,800$:
Line 4 in the table.

$74200 \leq x \leq 154800$: 

The equations for $T(x)$:

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For income between $30,650$ and $74,200$:
Line 3 in the table.
$30650 \leq x \leq 74200$: