I and Q Components in Communications Signals and Single Sideband

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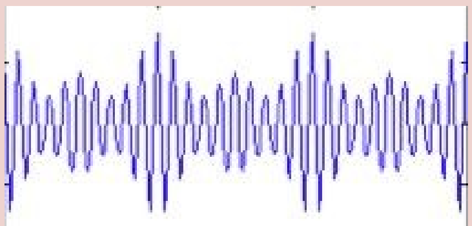
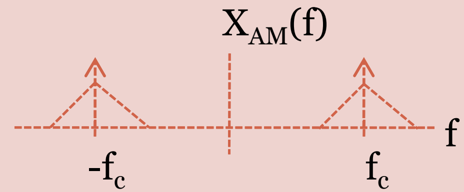
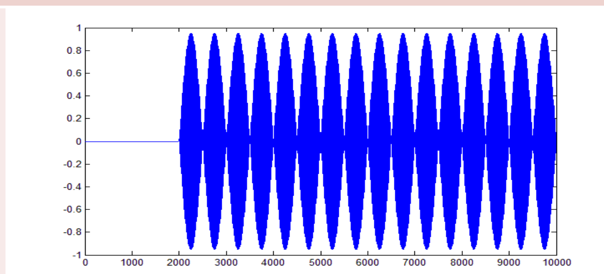
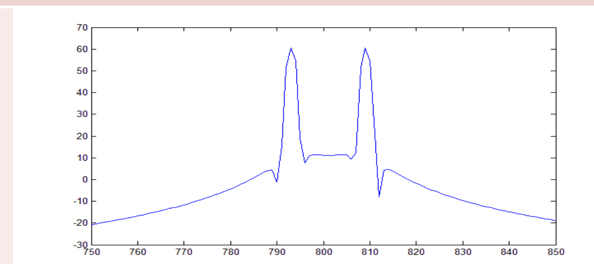
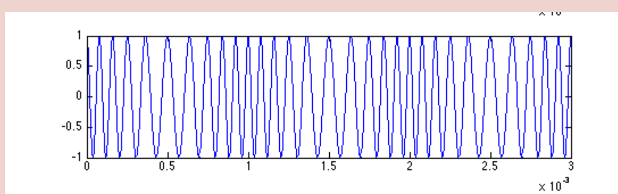
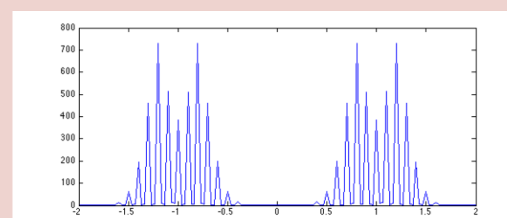
OVERVIEW

- Description of I and Q signal representation
- Advantages of using I and Q components
- Using I and Q to demodulate signals
- I and Q signal processing in the USRP
- Single Sideband (SSB)
- Processing I and Q components of a SSB signal in the USRP

Standard Representation of Communications Signals

Modulation	Time Domain	Frequency Domain
AM		
DSB		
FM		

Overview of I and Q Representation

- I and Q are the In-phase and Quadrature components of a signal.
- Complete description of a signal is:

$$x(t) = I(t) + jQ(t)$$

- x(t) can therefore be represented as a vector with magnitude and phase angle.
- Phase angle is not absolute, but relates to some arbitrary reference.

Overview of I and Q Representation

- In Digital Signal Processing (DSP), ultimate reference is local sampling clock.
- DSP relies heavily on I and Q signals for processing. Use of I and Q allows for processing of signals near DC or zero frequency.
  - If we use "real" signals (cosine) to shift a modulated signal to baseband we get sum and difference frequencies
  - If we use a "complex" sinusoid to shift a modulated signal to baseband we ONLY get the sum
  - This avoids problems with images

Overview of I and Q Representation

- Nyquist frequency is twice highest frequency, not twice bandwidth of signal.

  For example: common frequency used in analog signal processing is 455 kHz. To sample in digital processing, requires 910 kS/s. But if the signal bandwidth is only 10 kHz. With I & Q, sampling requires only 20 kS/s.

Overview of I and Q Representation

- I and Q allows discerning of positive and negative frequencies.
  - If : $H(f) = a + jb$
  - Then: $H(-f) = a - jb$

Overview of I and Q Representation

Representing familiar characteristics of a signal with I and Q:

- Amplitude: $A(t) = \sqrt{I^2(t) + Q^2(t)}$
- Phase: $\phi(t) = \tan^{-1}\left(\frac{Q(t)}{I(t)}\right)$
- Frequency:

$$f(t) = \frac{\partial \phi(t)}{\partial t} = \frac{I(t)\frac{\partial Q(t)}{\partial t} - Q(t)\frac{\partial I(t)}{\partial t}}{I^2(t) + Q^2(t)}$$

DEMODULATION

- AM: $x(t) = \sqrt{i^2(t) + q^2(t)}$
- SSB: $x(t) = i(t)$
- FM: $x(t) = \left(\frac{1}{\Delta t}\right)\tan^{-1}\left[\frac{i(t)q(t-1) + q(t)i(t-1)}{i(t)i(t-1) - q(t)q(t-1)}\right]$
- PM: $x(t) = \tan^{-1}\left[\frac{q(t)}{i(t)}\right]$

Overview of I and Q Representation

- The traditional FM equation:

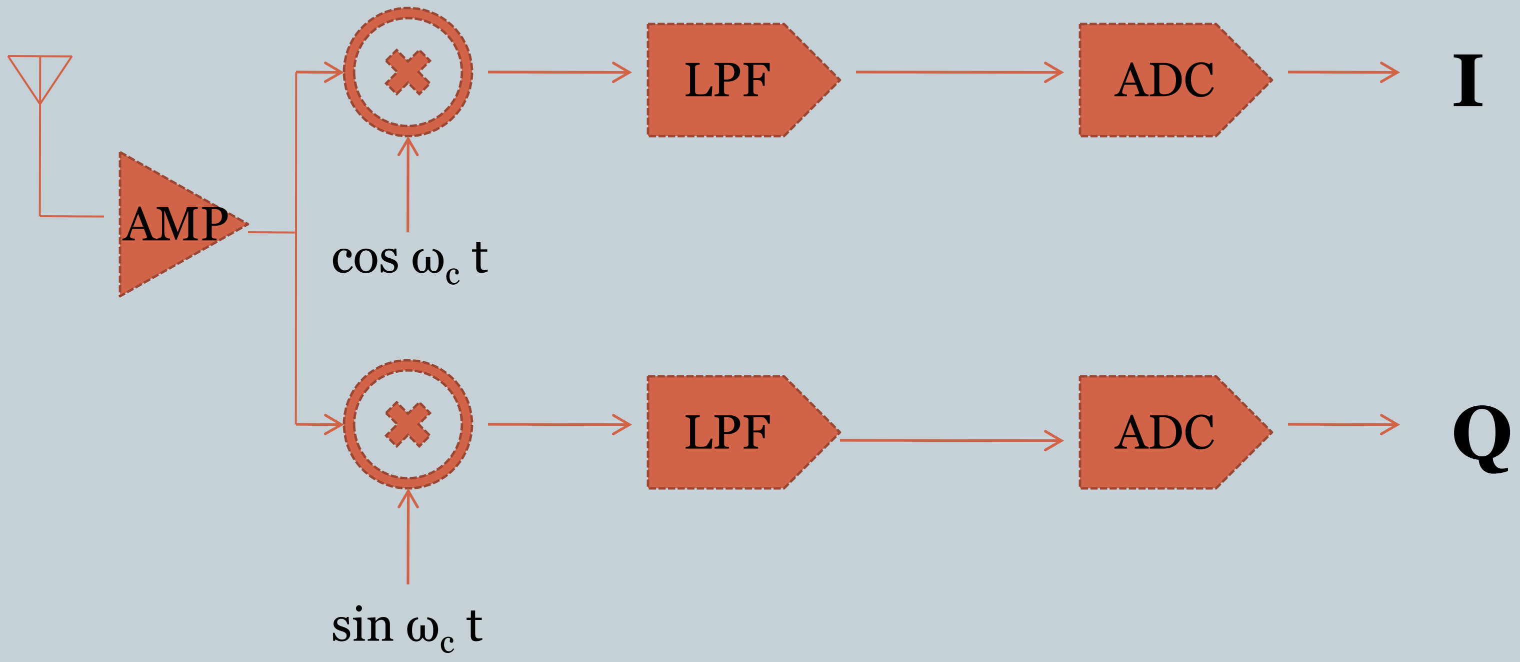
$$x_{FM}(t) = \cos(\omega_c t + k\int x_m(t)dt)$$

- The analytic equation:

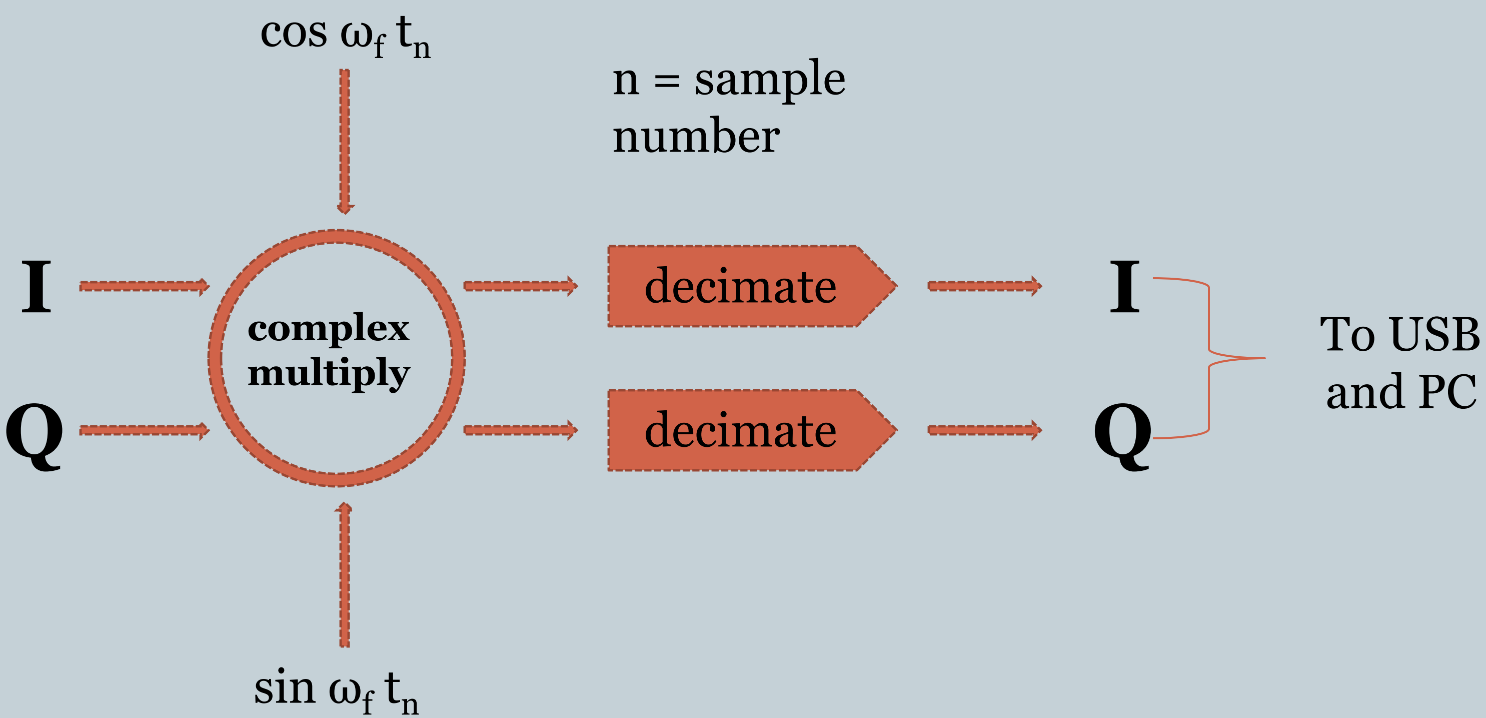
$$x_{FM}(t) = I(t)\cos(\omega_c t) + jQ(t)\sin(\omega_c t)$$

- Modulation and Demodulation methods are different when I and Q representation is used

USRP DAUGHTER BOARD

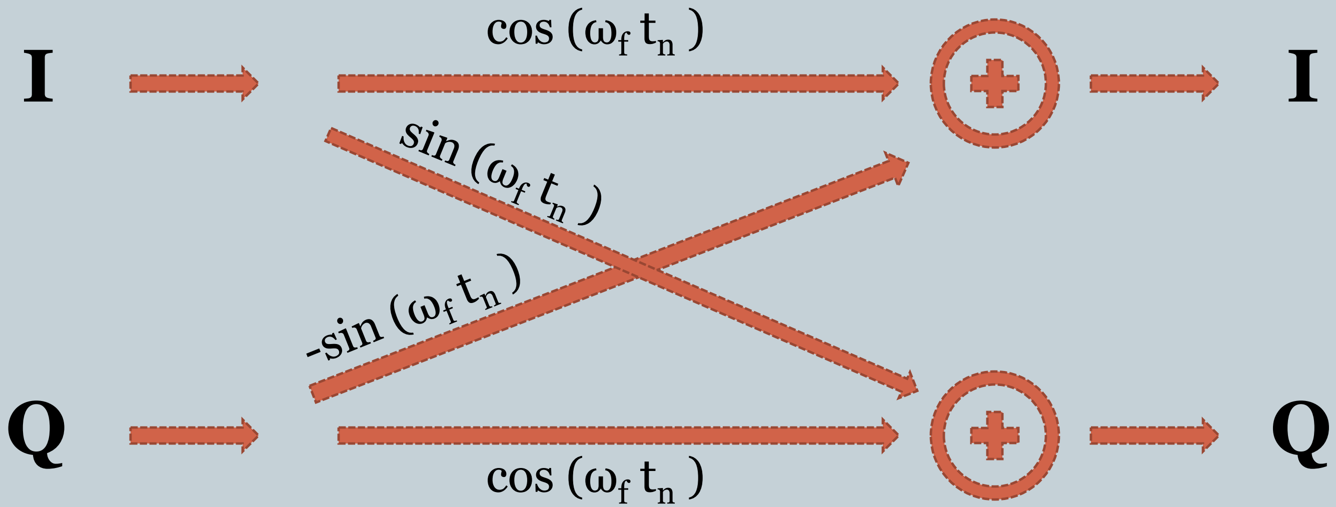


FPGA

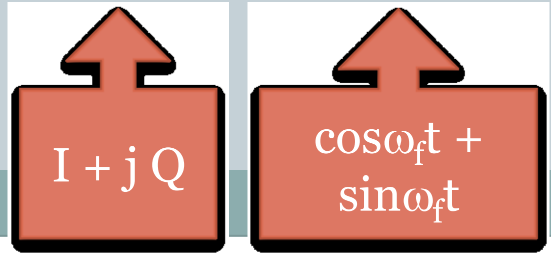


cos $\omega_f t_n$
n = sample number
I
Q
complex multiply
decimate
decimate
I
Q
To USB and PC
sin $\omega_f t_n$

Complex Multiply



$$(A + j\,B) * (C + j\,D) = AC - BD + j\,(BC + AD)$$

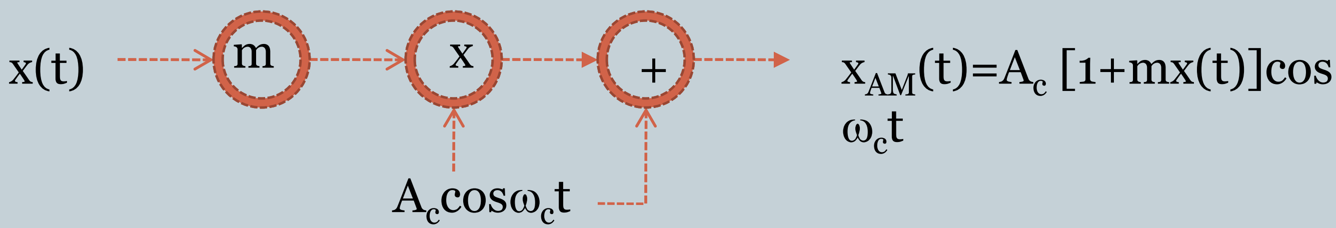


Sideband Modulation

- Where's the intelligence?
  - A signal carries useful information only when it changes.
  - Change of **ANY** carrier parameter produces sidebands.
  - The intelligence or information is in the sidebands.
- Why not just send the sidebands or just a sideband?

AM Review

- AM review:
  - Carrier is modulated by varying amplitude linearly proportional to intelligence (baseband) signal amplitude.
- Block Diagram

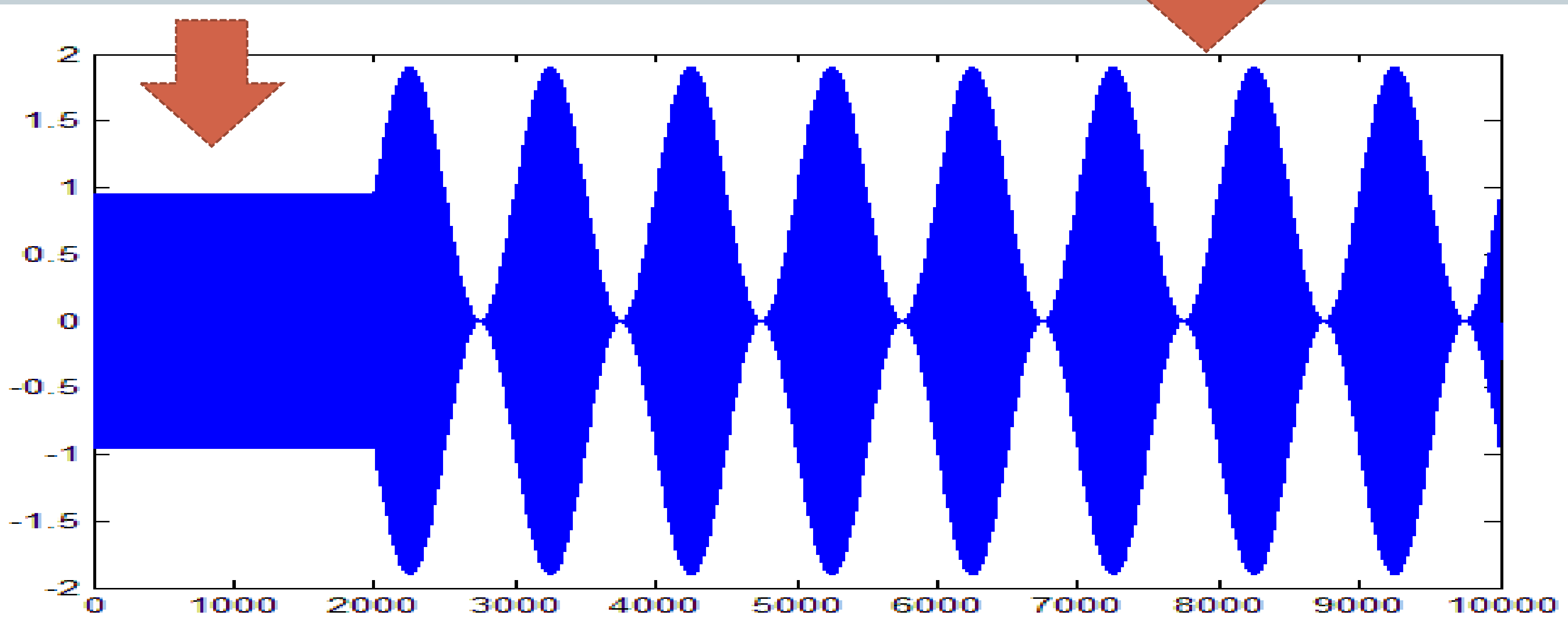


AM: Time Domain

- AM in the Time Domain

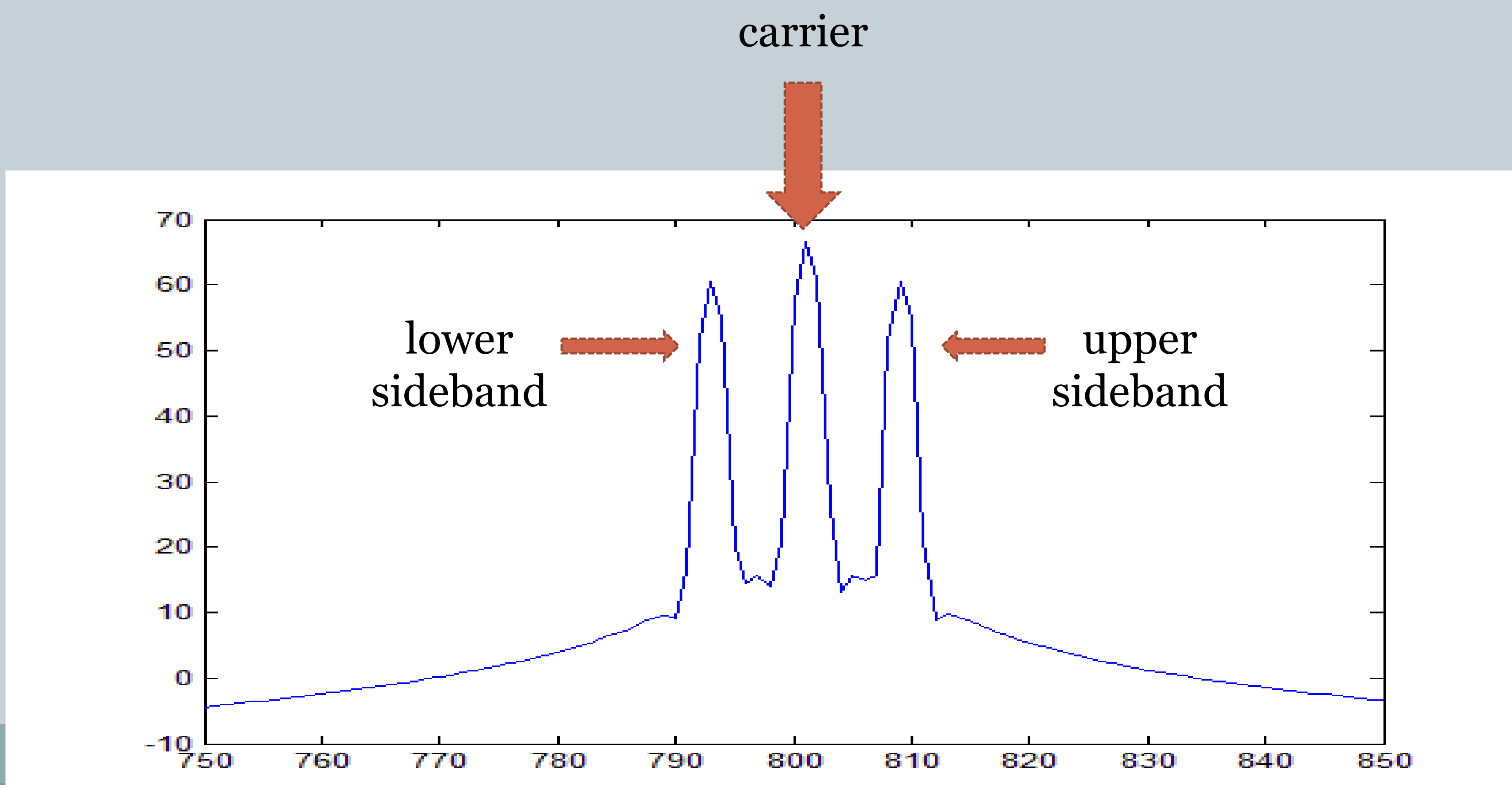
100% modulated carrier

Unmodulated carrier



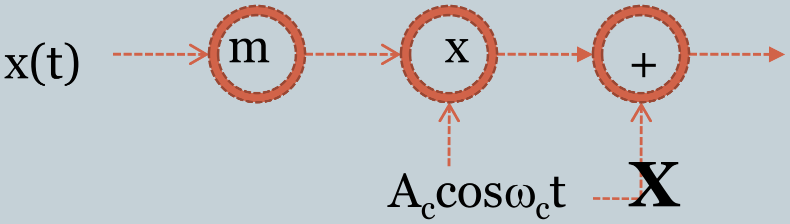
AM: Frequency Domain

- AM in the Frequency Domain



Double Sideband Modulation (DSB)

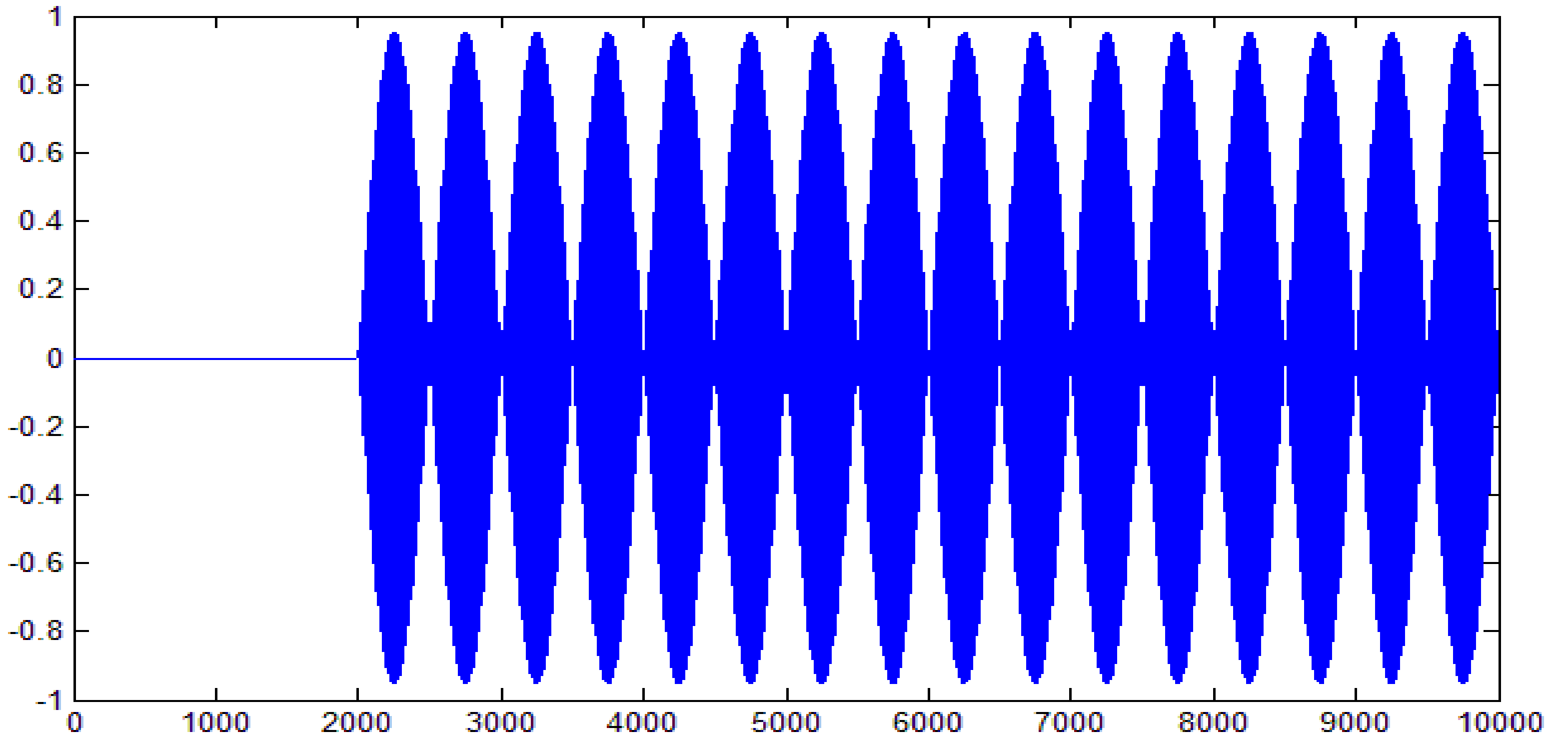
- Let’s just transmit the sidebands



$x_{DSB}(t) = A_c * m * x(t) * \cos\omega_c t$

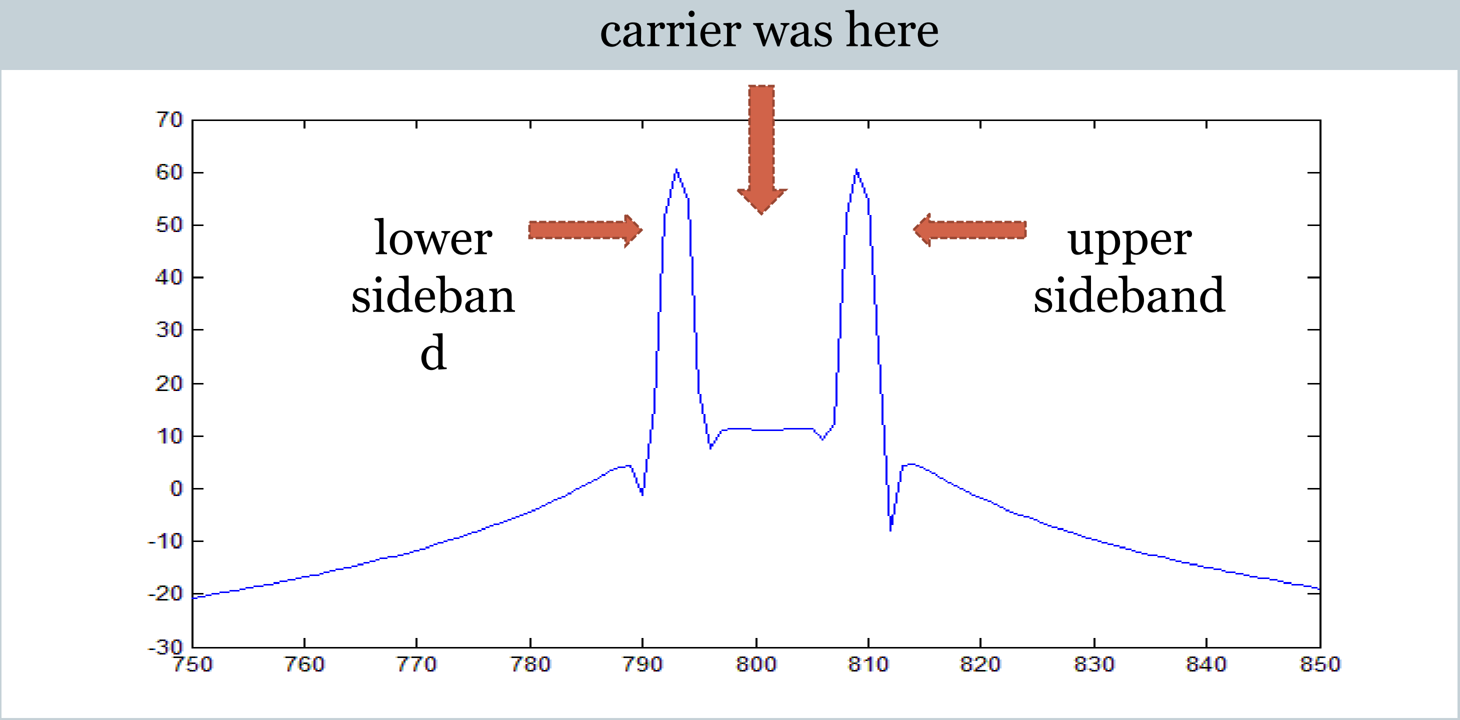
DSB: Time Domain

- Double Sideband in the Time Domain

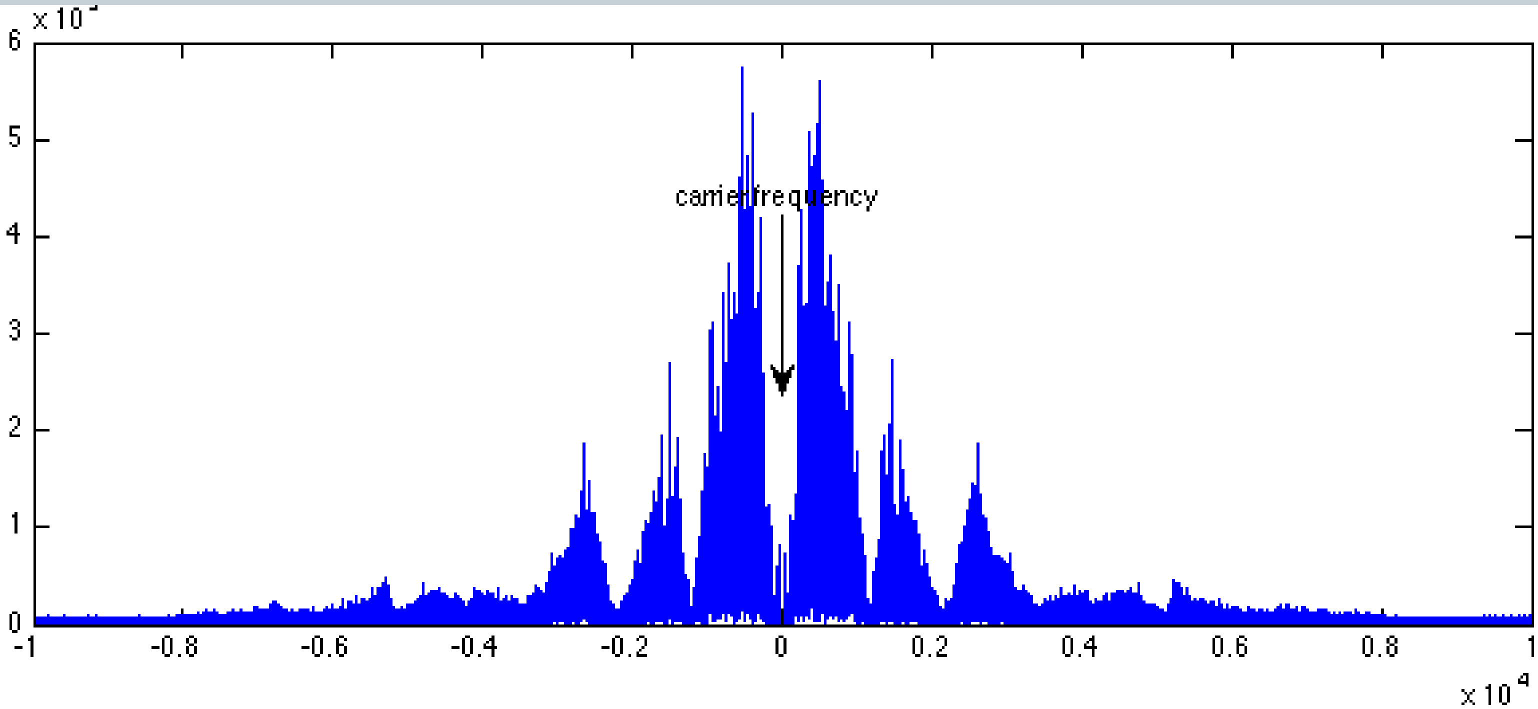


DSB: Frequency Domain

- Double Sideband in the Frequency Domain

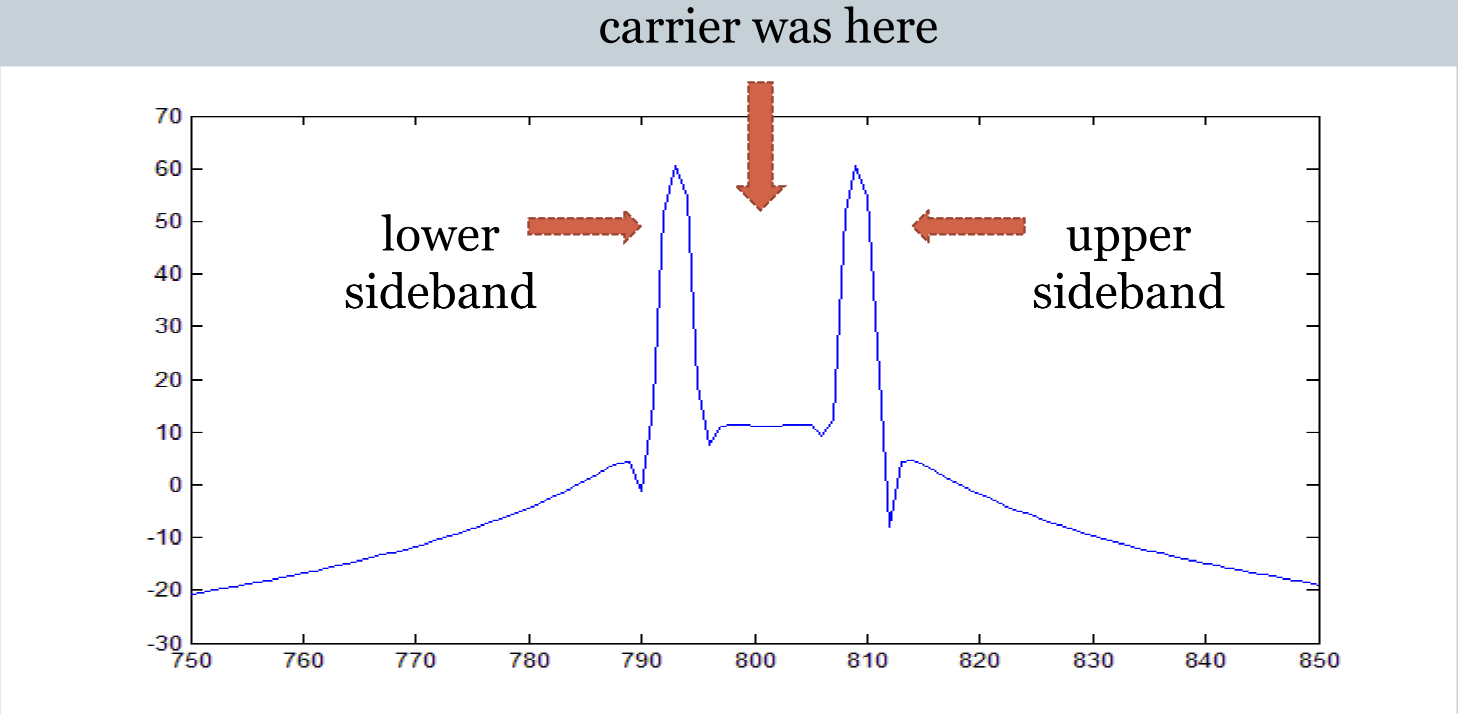


Example of a DSB Signal



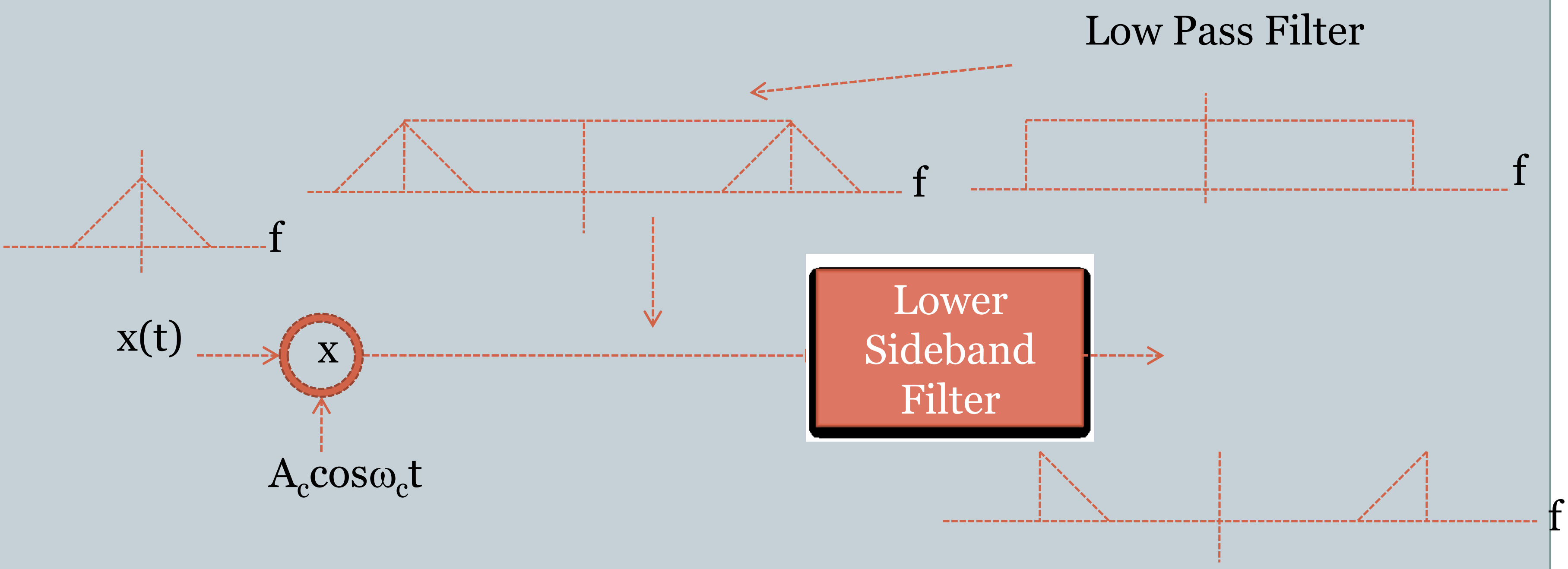
DSB Spectrum

- Note: the upper and lower sidebands are the same
- Do we need both of them?

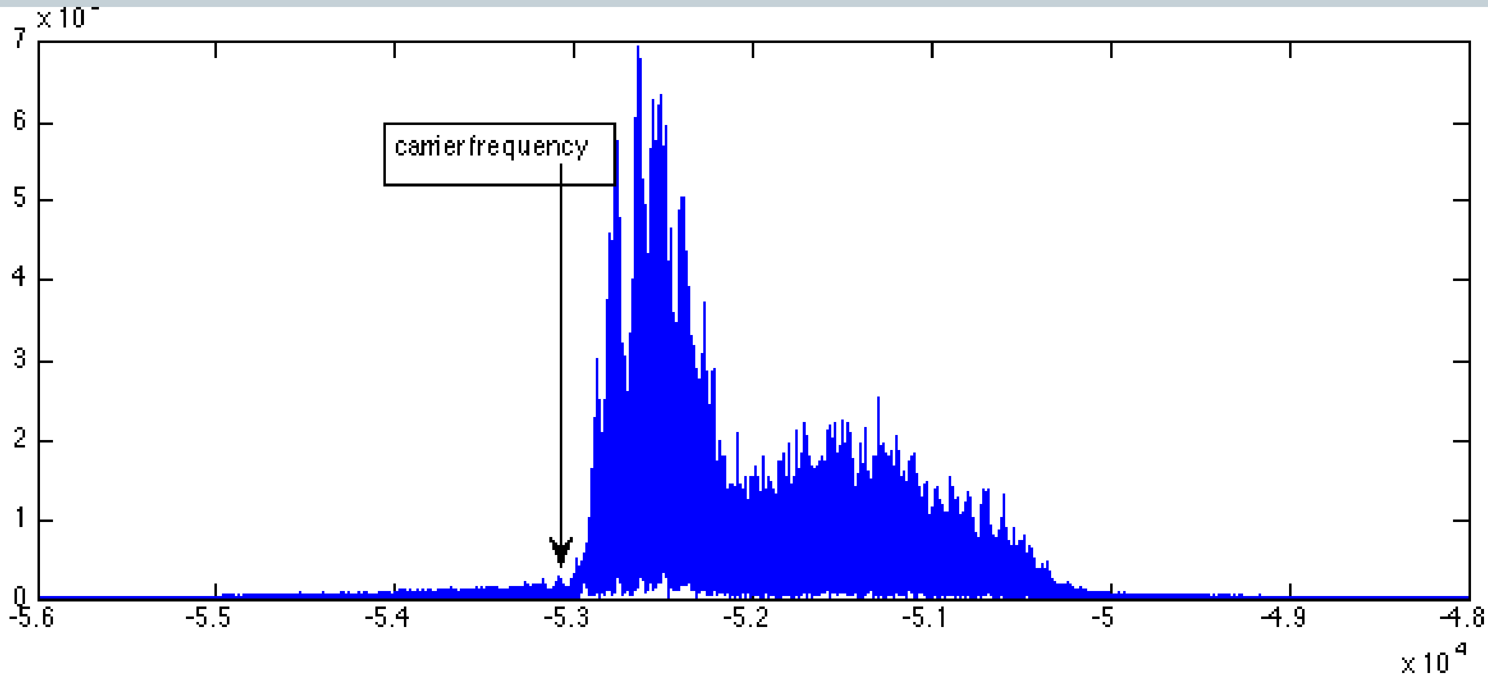


SSB Signals

- A sideband signal is obtained by adding a sideband filter to capture the upper or lower sideband.



Example of a USB Signal

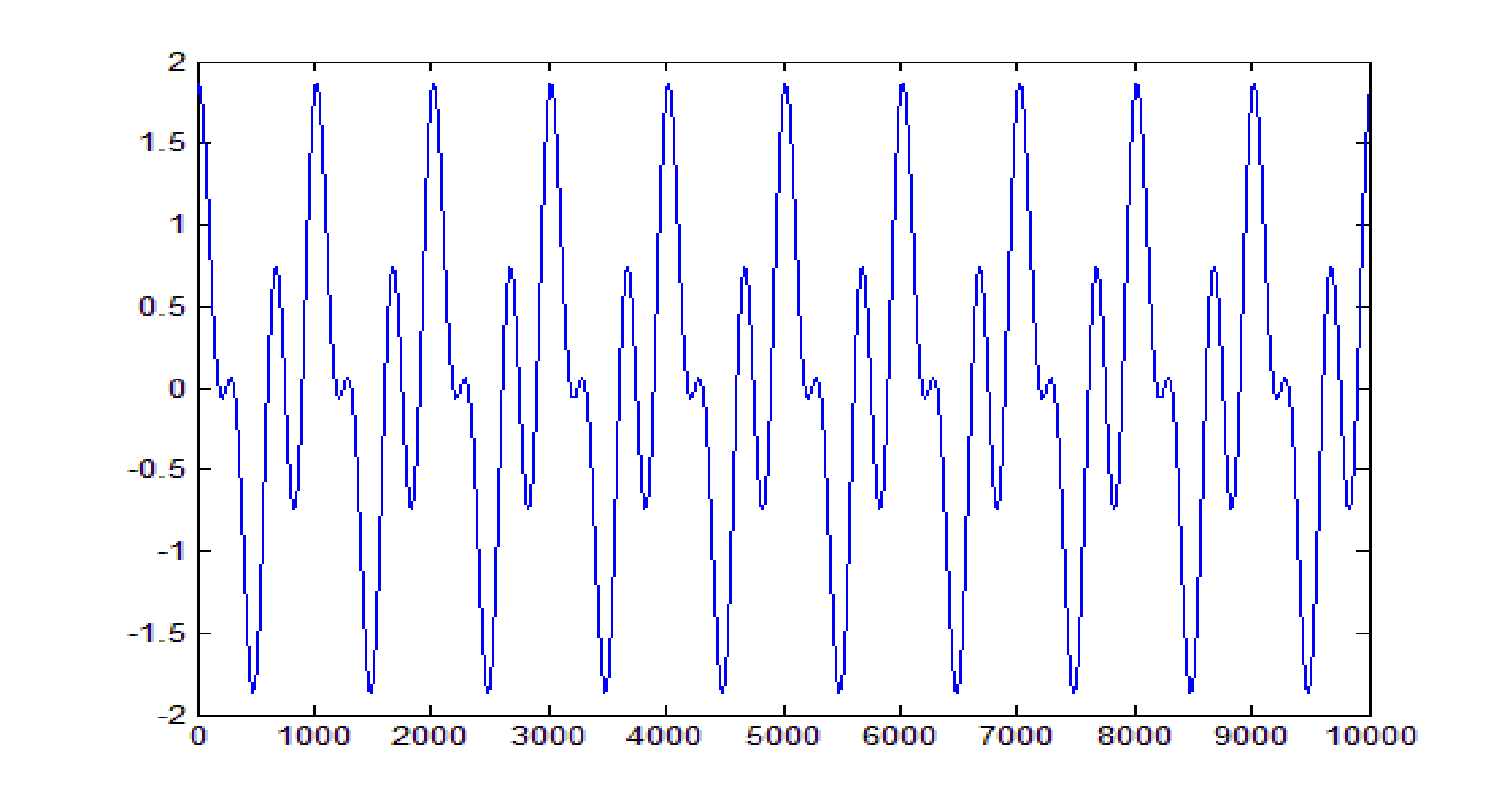


Comparison of DSB and SSB

- Power: SSB requires half of the power of DSB
- Bandwidth: SSB requires half of the bandwidth of DSB
- Complexity: SSB modulators/demodulators are more complex

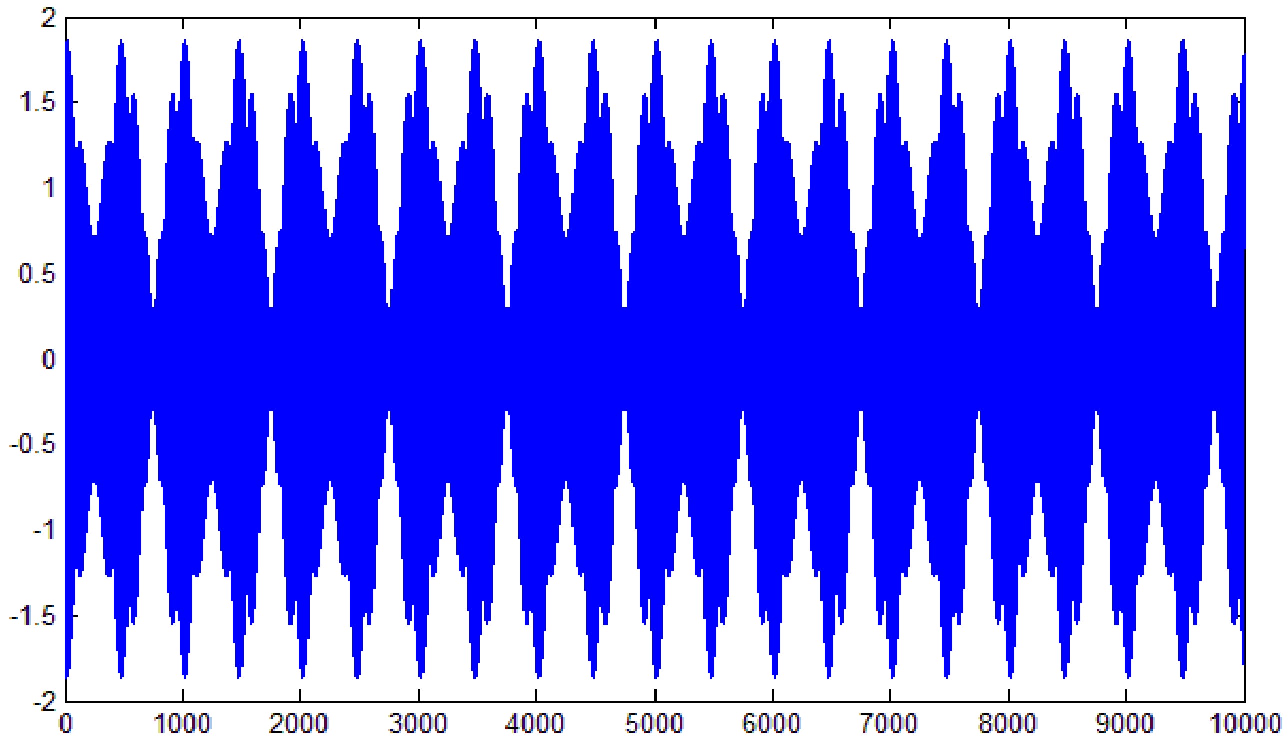
SSB Example

- Start with arbitrary waveform in baseband:

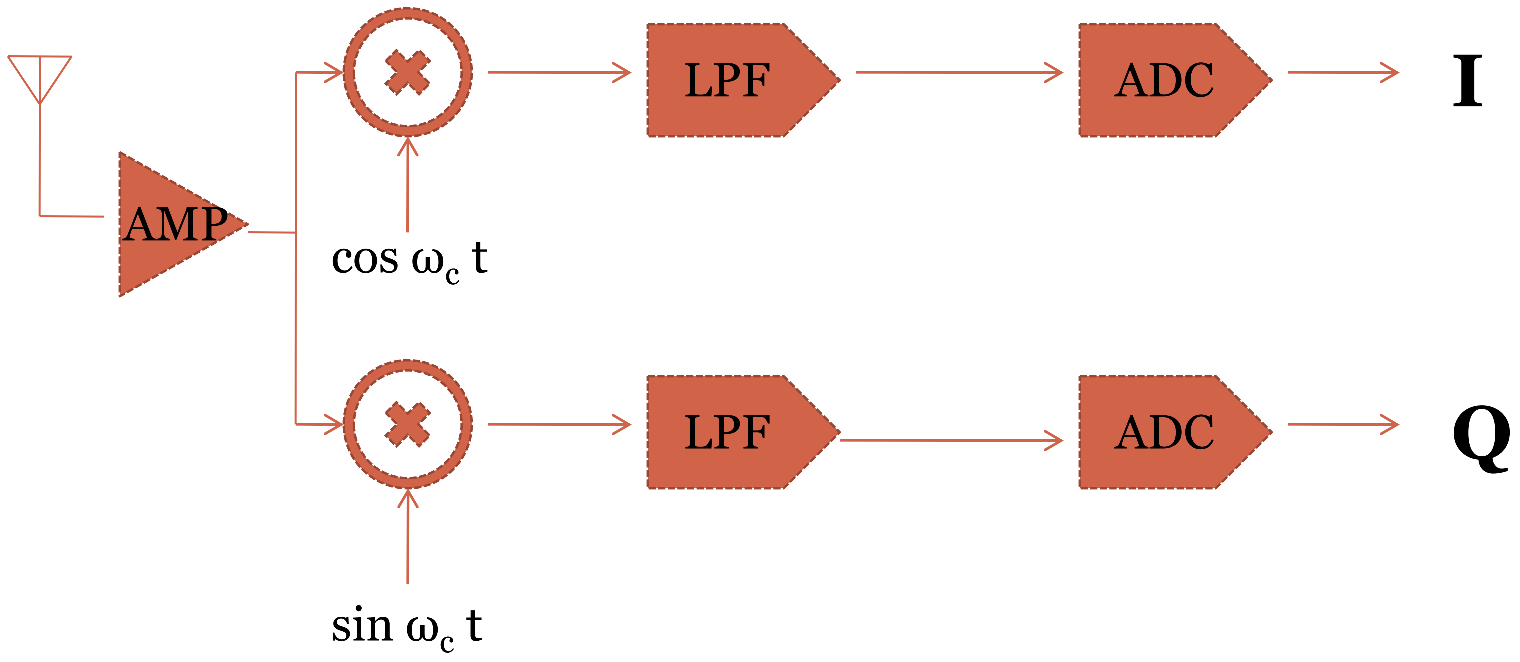


SSB Example

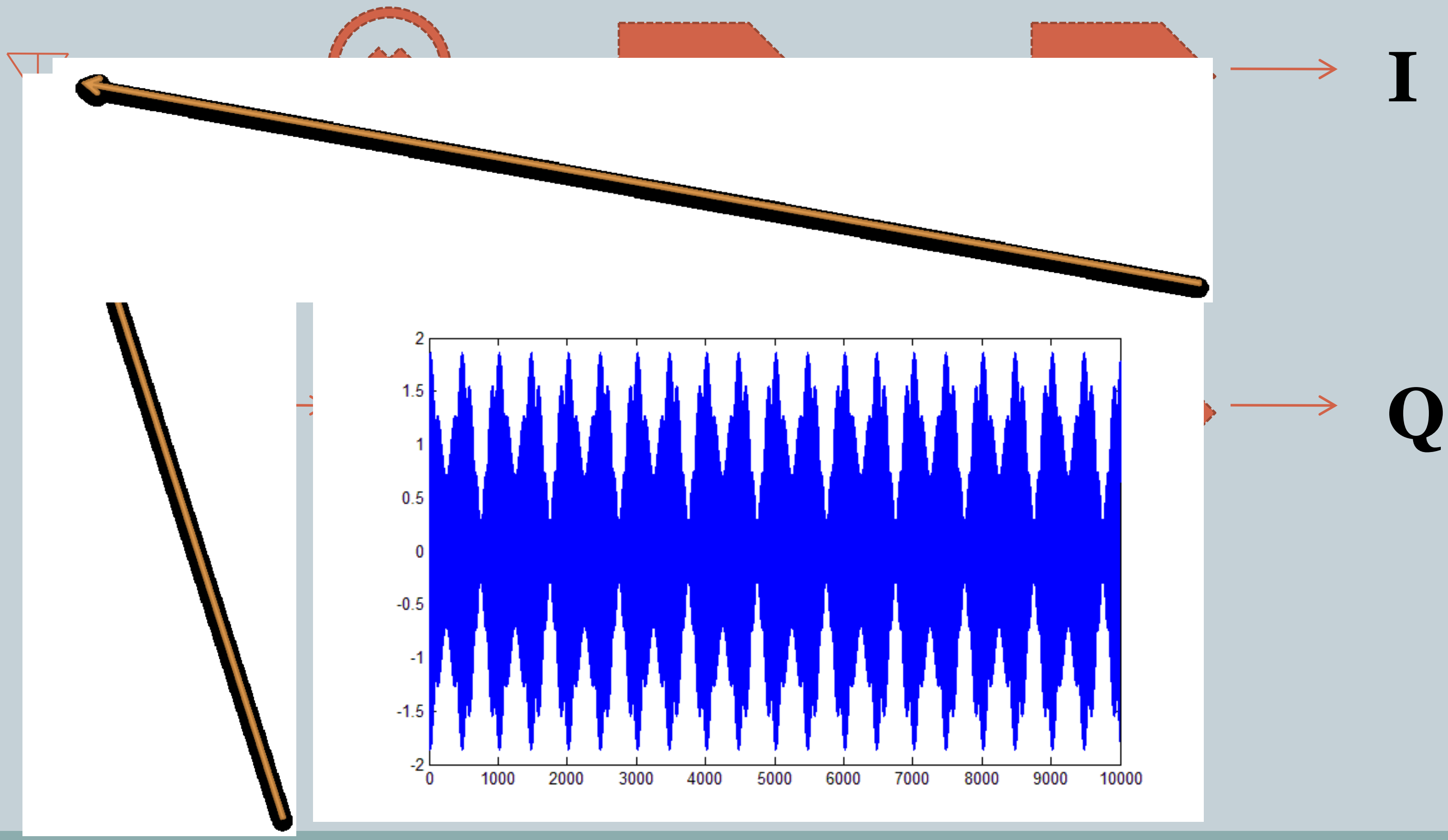
- Modulate as Upper Sideband Signal:



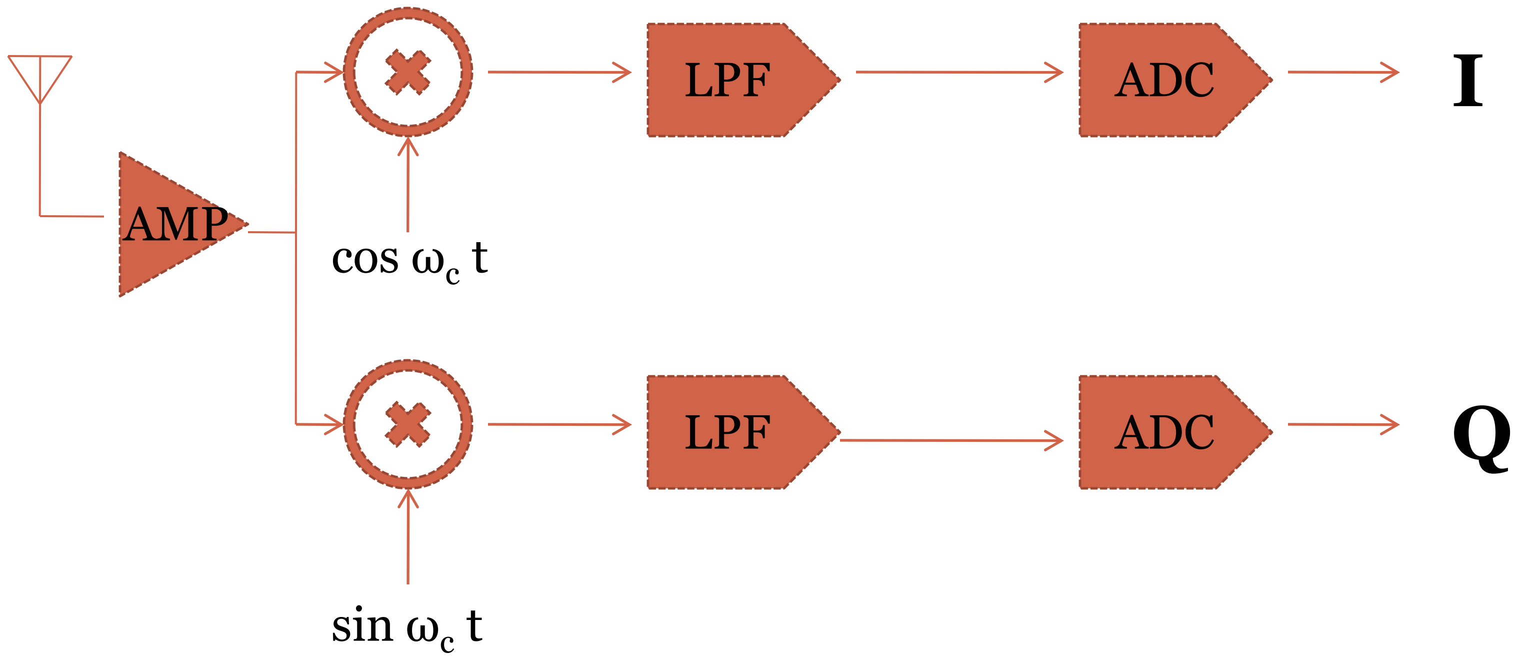
SSB Example



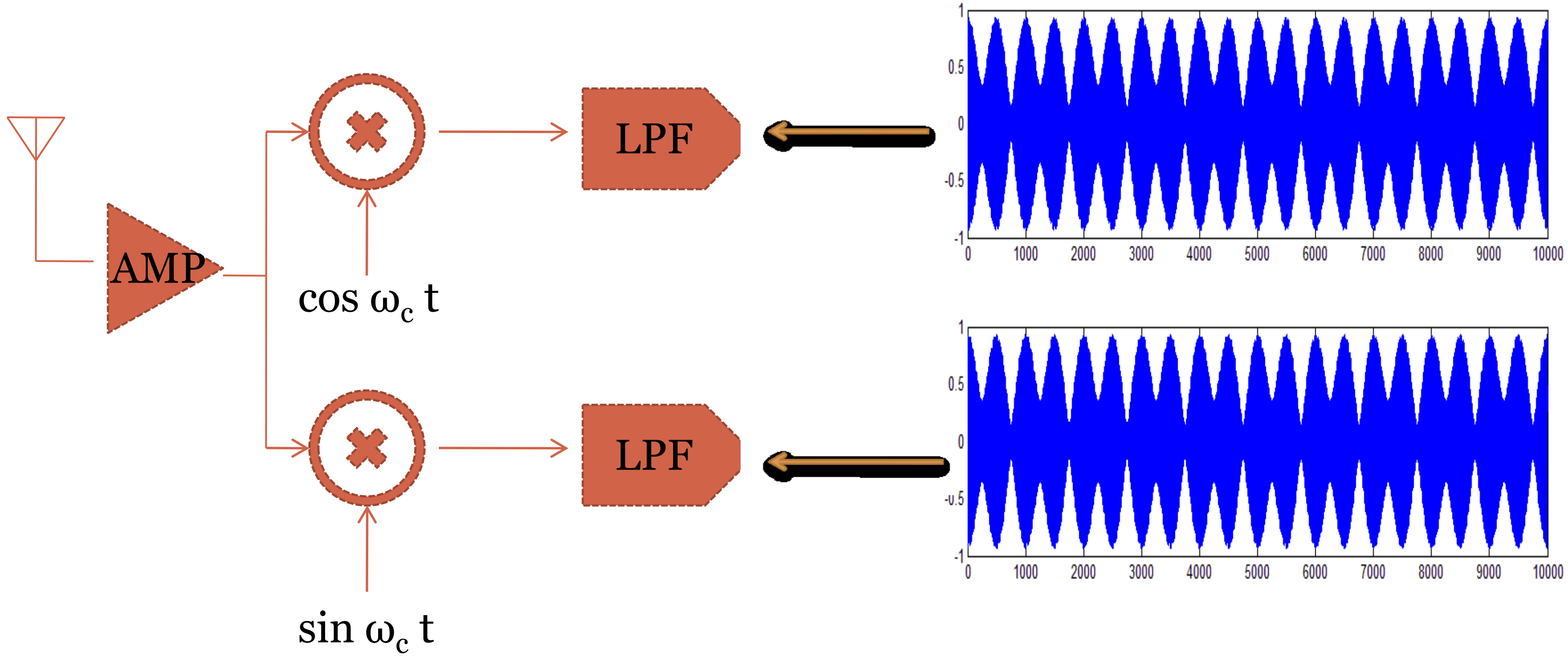
SSB Example


I
Q
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000

SSB Example

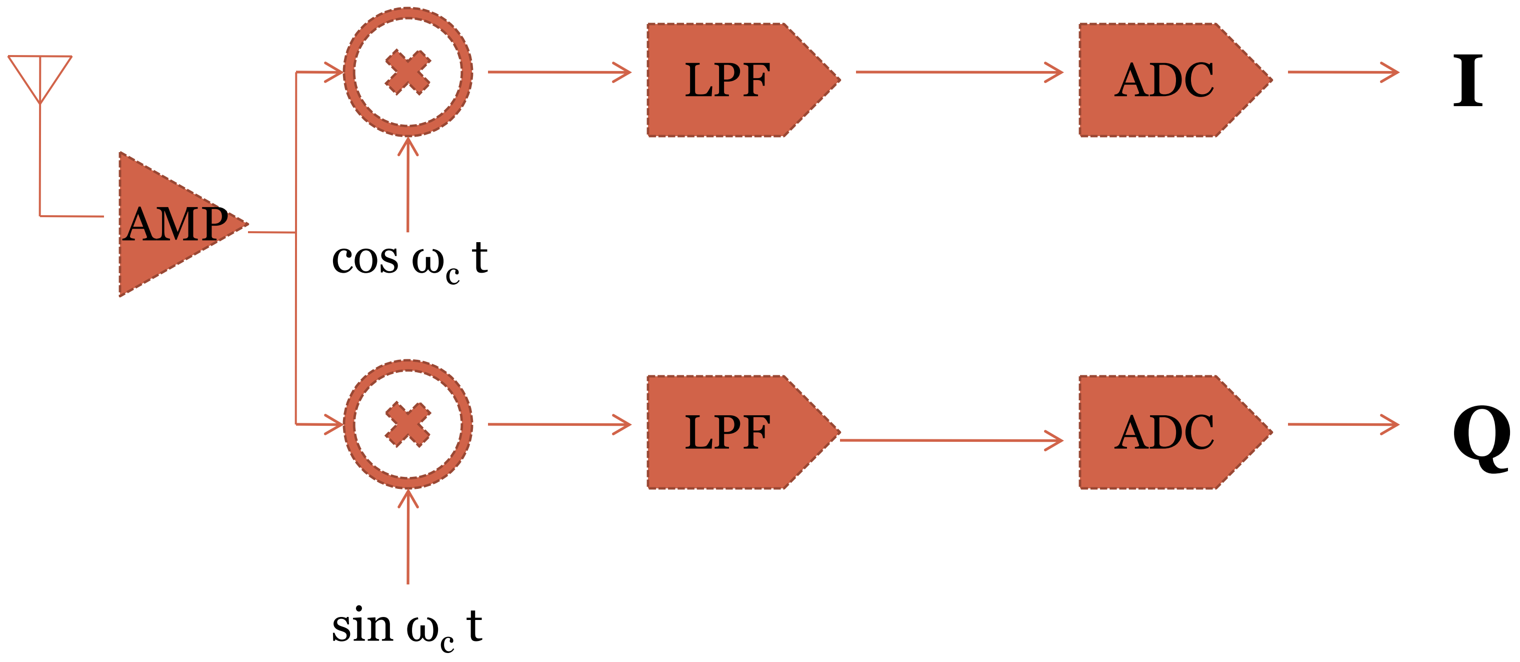


SSB Example

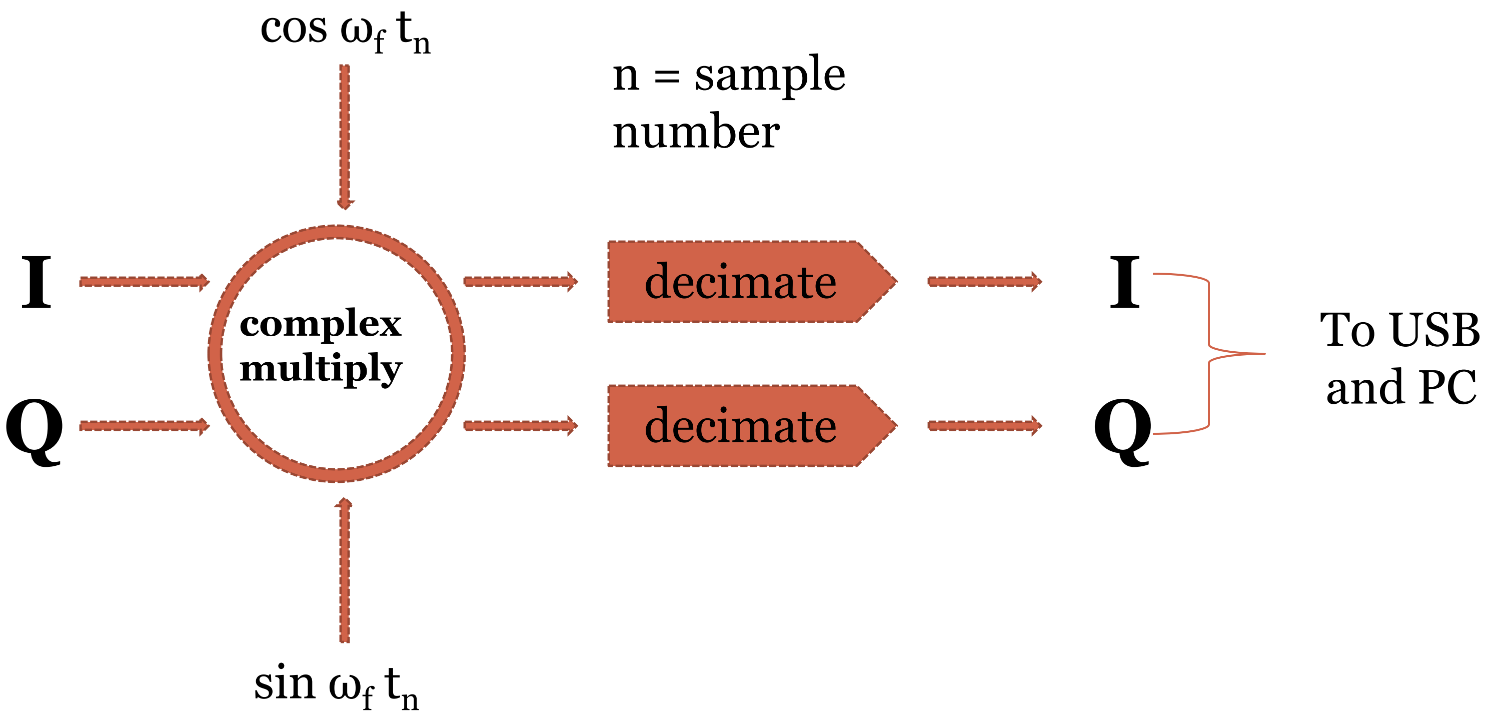


AMP
cos ω_c t
sin ω_c t
LPF
LPF
1
0.5
0
-0.5
-1
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000

SSB Example

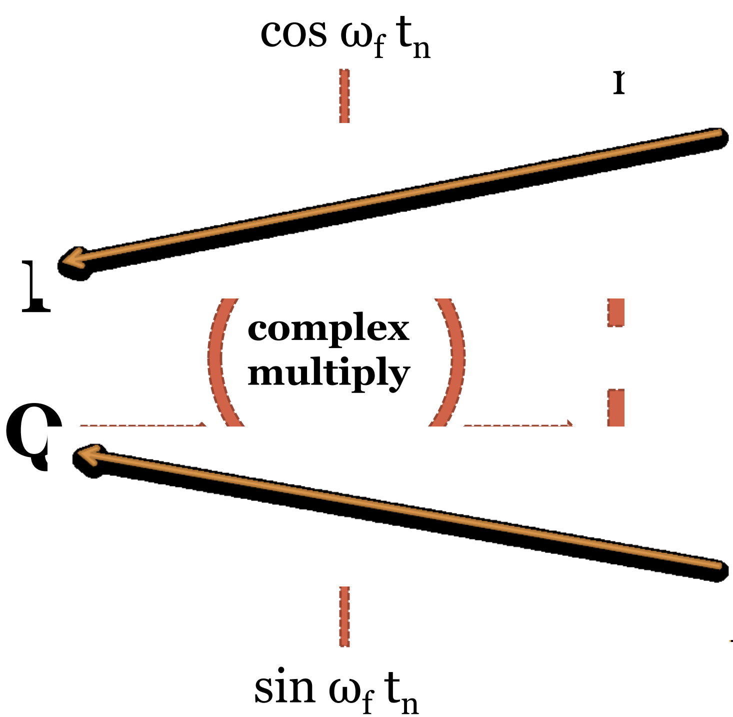


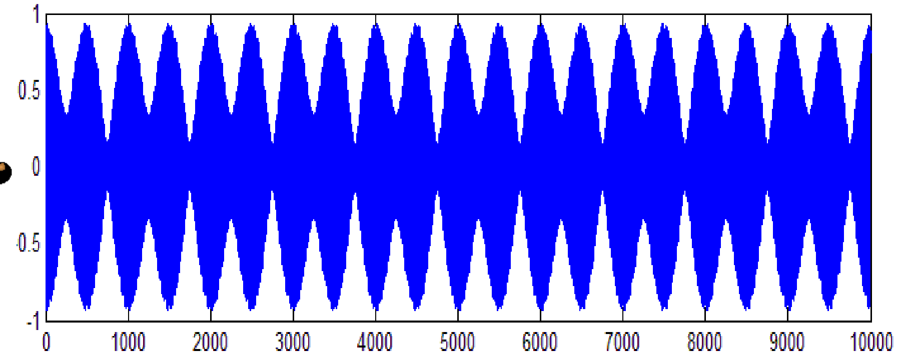
SSB Example

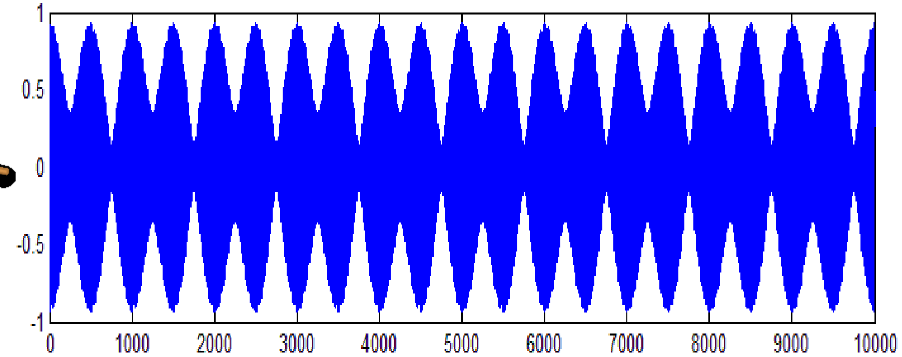


cos ωf tn
n = sample number
I
Q
complex multiply
decimate
decimate
I
Q
To USB and PC
sin ωf tn

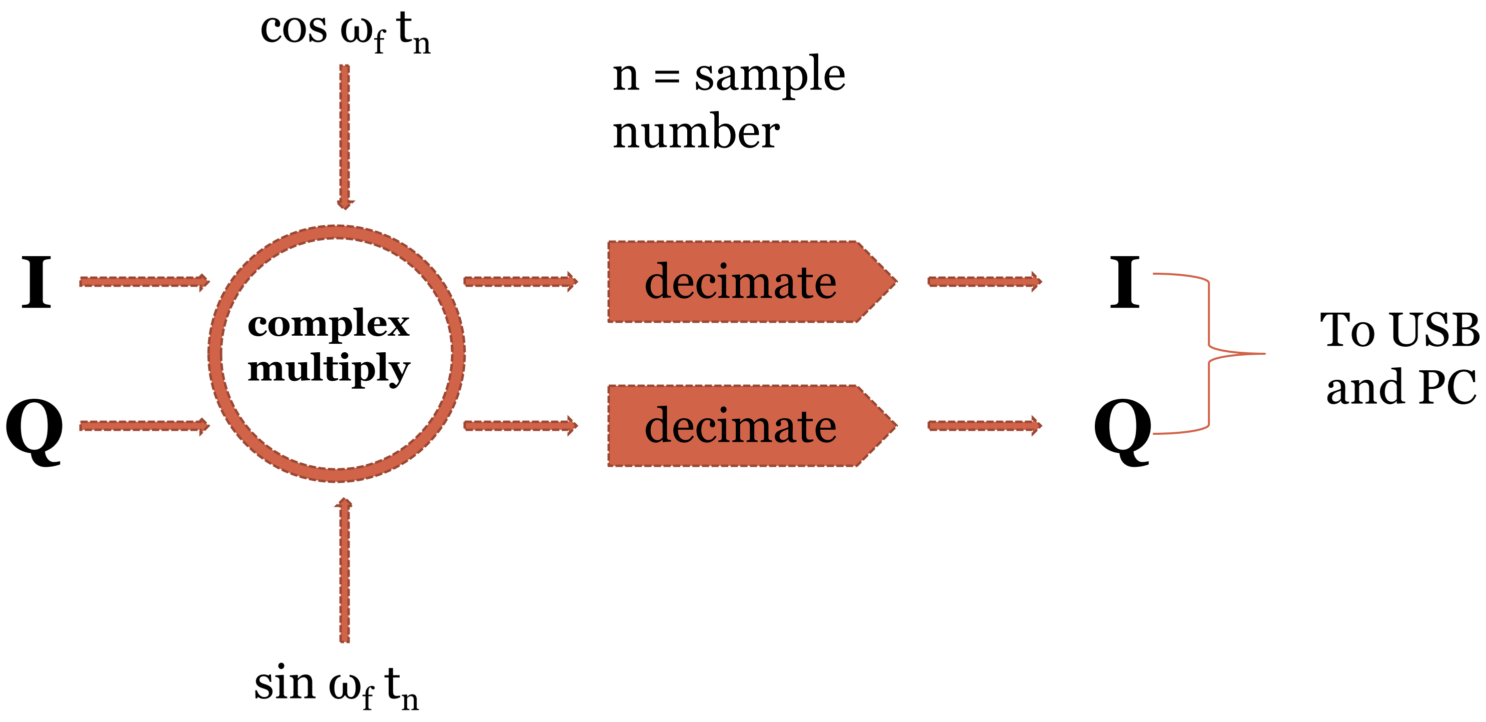
SSB Example







SSB Example



cos ωf tn
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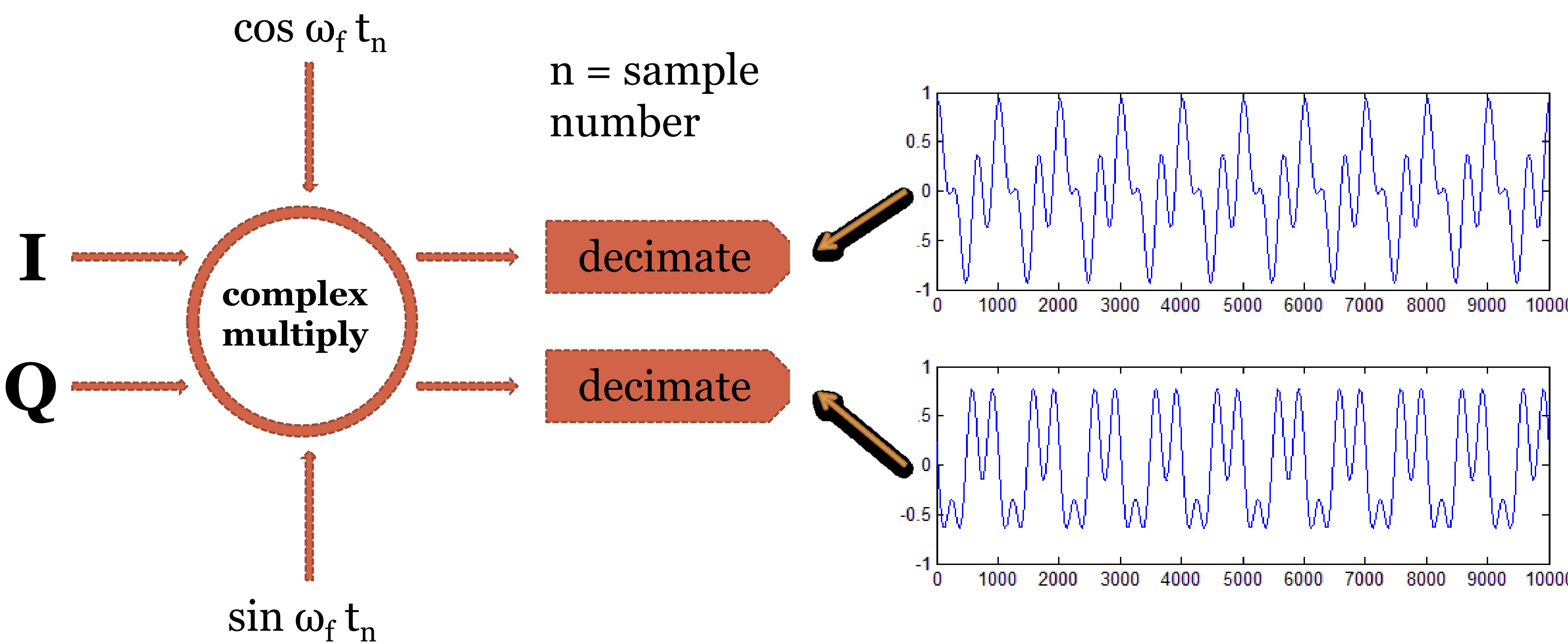
sin ωf tn

SSB Example



$\cos \omega_f t_n$
n = sample number
I
Q
complex multiply
decimate
decimate
$\sin \omega_f t_n$
1
0.5
0
.5
-1
0 1000 2000 3000 4000 5000 6000 7000 8000 9000 10000
1
.5
0
-0.5
-1
0 1000 2000 3000 4000 5000 6000 7000 8000 9000 10000

Practical SSB Reception

- Example above assumed no other signals on the band and perfectly synchronized oscillators
- Need to isolate (filter) the signal of interest and deal with oscillators slightly out of sync
- GRC tutorial demonstrates Weaver's Method of demodulating SSB that solves these problems