Discrete Mathematics Projects

Prof. Silvia Fernández **Discrete Mathematics** **Math 513B, Spring 2007**

Project 1. Game Theory

Designing interesting games and/or finding winning strategies for known games. Describe the game in terms of graphs, what are you trying to achieve or avoid? Some possibilities are:

1. In the game of Dots and Boxes you are trying to achieve as many 1×1 squares in the graph whose vertices are lattice points and the edges join any two consecutive horizontal or vertical vertices. Note that the 1×1 squares in this graph are actually the circuits of length 4. In other words, players take turns at choosing one edge from the graph determined by the grid and the goal of the game is to have as many cycles of length 4 as possible. Strategies for this game have been studied. Can you explain some of these strategies in terms of graph theory concepts/theorems?

2. The game of Dot and Boxes (above) can be generalized (modified) as follows.

   Replace the grid by any graph G and choose a subgraph H of G. As before players take turns selecting edges from G. There can be several goals in this game. Some examples are:

   (a) The player that accumulates the most copies of H wins.
   (b) The player that first completes a copy of H wins.
   (c) The player who first completes a copy of H losses.

   This way you can create your own game(s) with an analysis of game strategies.

3. What games that you know can be studied in terms of graphs? Is Tic-Tac-Toe an example of this? What is the graph? What are you trying to avoid? You can also study Tic-Tac-Toe generalizations.

4. Find out how the Game of Dim is played and study it from the graph theory point of view.

Project 2. Number Theory

1. Survey of arithmetic functions $\sigma, \tau, \mu, \varphi$ and how to compute them. Combinatorial proofs (See Ex. 7.1.32 p. 424.)

2. Understand divisibility criteria. Develop divisibility criteria for "nontraditional" primes, like 7 or 13. Can you explain why these are not usually mentioned in the regular literature?

Project 3. Scheduling Problems

1. Look for explicit scheduling examples in your life. Here are some questions to consider.

2. When organizing a conference or event, how do you schedule the talks according to participant and room restrictions? How are final exams, spelling bee competitions, sports competitions scheduled at your school? How do you make sure that all or must participants will be able to attend their events or exams?

3. How many teachers' committees are there in your school and how are they formed? When are regular faculty meetings scheduled? How are classes assigned to you? How are classes scheduled?

4. Look at Ex. 8 p. 744.

Use different techniques and levels of difficulty: weighted graphs, SDRs, matchings, chromatic polynomials.

Project 4. Power in games

Look for any kind of real life examples where some kind of vote takes place. Model and determine the power that each involved party has using the Shapley-Shubik power index. Here are some possibilities.

1. Find out how much power each state has in the US presidential elections according to the Shapley-Shubik power index.

2. Are there student elections at your school? Do you vote on certain school decisions? How much power do you have in an election where each department has one vote but some departments are larger than others? What kind of majority is needed?

Project 5. Matchings, SDRs, and Stable Marriage Problems

How to form teams in your class according to classmates preferences (lists of preferences are provided by each student)? Look for other variations on this situation.

Project 6. Pascal's Triangle and Binomial Coefficients

Study Pascal's triangle and look for any kind of patterns and binomial coefficient identities. Do the same for Pascal's triangle mod n. What is Pascal's tetrahedron? How can you use it in algebraic expansions, that is, is there something like the binomial theorem for Pascal's tetrahedron?

Project 7. Algebraic representations of graphs

Study the adjacency matrix of a graph. How can you find the number of edges, the degrees, the number of triangles, etc., without drawing the graph. Just by using the entries of the matrix.

Project 8. Latin squares

Study Latin squares and their relationship to Sudoku. How many Latin squares are there of a fixed size under certain given restrictions? Come up with easier but still interesting Sudoku variations.