Thu, Aug 7, 2014 **MATH 280, Quiz 6** Prof. V. Panferov

Name (print): Solutions.

Each problem is worth 2 points. Show all your work.

1. Solve by using the substitution $y' = u$:

$$y'' + 2y(y')^3 = 0.$$

$$y' = u \ ; \quad y'' = \frac{du}{dy}\frac{dy}{dx} = \frac{du}{dy}u$$

$$u\frac{du}{dy} + 2yu^3 = 0$$

$$\frac{du}{u^2} = -2y\,dy$$

$$-\frac{1}{u} = \int\frac{du}{u^2} = -\int 2y\,dy = -y^2 - c_1$$

$$\frac{1}{u} = y^2 + c_1$$

$$u = \frac{1}{y^2 + c_1}$$

$$\frac{dy}{dx} = \frac{1}{y^2 + c_1}$$

$$(y^2 + c_1)\,dy = dx$$

$$\int(y^2 + c_1)\,dy = \int dx = x - c_2$$

$$\frac{y^3}{3} + c_1 y = x - c_2$$

$$x = \frac{y^3}{3} + c_1 y + c_2$$

Please turn over...

2. A mass weighing 4 lb is attached to a spring whose constant is 2 lb/ft. The medium offers a damping force with coefficient β= 1 lb s/ft. The mass is initially released from a point 1 ft above the equilibrium position with a downward velocity of 8 ft/s. Determine the position of the mass $x(t)$ for $t > 0$. *Hint: To determine the mass use the value of acceleration of gravity 32 ft/s^2.*

$$m = \frac{4\ lb}{32\ ft/s^2} = \frac{1}{8}\ [slugs] \quad ; \quad k = 2\ lb/ft$$

$$\beta = 1\ lb\,s/ft$$

$$\frac{1}{8}\ddot{x} + \dot{x} + 2x = 0$$

$$x(0) = 1,\ \dot{x}(0) = -8$$

$$\ddot{x} + 8\dot{x} + 16x = 0$$

$$m^2 + 8m + 16 = 0$$

$$(m+4)^2 = 0$$

$$m = -4 \quad (\text{multiplicity } 2)$$

$$x(t) = c_1 e^{-4t} + c_2 t e^{-4t}$$

$$x(0) = c_1 = 1$$

$$\dot{x}(0) = \left(-4c_1 e^{-4t} + c_2 e^{-4t} - 4c_2 t e^{-4t}\right)\Big|_{t=0}$$

$$= -4c_1 + c_2 = -8$$

$$c_2 = -4$$

$$x(t) = e^{-4t} - 4t e^{-4t}$$

Please turn over...

3. Find the eigenvalues and the eigenfunctions of the boundary-value problem:

$$y'' + \lambda y = 0, \quad y(0) = 0, \quad y(\pi) = 0.$$

Hint: check only $\lambda > 0$, there are no eigenvalues with $\lambda \leq 0$.

$\lambda > 0$: $\quad y(x) = c_1 \cos \alpha x + c_2 \sin \alpha x$

$\lambda = \alpha^2 \quad y(0) = c_1 = 0$

$y(\pi) = c_2 \sin(\alpha\pi)$

$\alpha = n, \quad n = 1, 2 \dots$

$\lambda_n = n^2$ - eigenvalues

$y_n(x) = \sin(nx)$ - eigenfunctions.