Tue, Aug 5, 2014 MATH 280, Quiz 5 Prof. V. Panferov

Name (print): Solutions.

Each problem is worth 2 points. Show all your work.

1. Find the general solution of the equation $x^2y'' + 5xy' + 4y = 0$.

Cauchy-Euler equation: Try $y = x^m$;

$$m(m-1) + 5m + 4 = 0$$
$$m^2 + 4m + 4 = 0$$
$$(m+2)^2 = 0$$
$$m = -2 \quad \text{(multiplicity 2)}$$

Solution (case II) $y(x) = C_1 x^{-2} + C_2 x^{-2} \ln x$.

2. Find the general solution using variation of parameters: $y'' + y = \sec x$.

Homogen.: $y'' + y = 0$

$y(x) = C_1 \cos x + C_2 \sin x$.

$$W(x) = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1.$$

$$W_1 = \begin{vmatrix} 0 & \sin x \\ \sec x & \cos x \end{vmatrix} = -\frac{\sin x}{\cos x} = -\tan x.$$

$$W_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \sec x \end{vmatrix} = 1.$$

$$u_1' = \frac{W_1}{W} = -\tan x \quad ; \quad u_2' = \frac{W_2}{W} = 1$$

$$u_1 = -\int \tan x = -\int \frac{\sin x}{\cos x} dx = \int \frac{d\cos x}{\cos x} = \ln|\cos x| + C_1$$

$$u_2 = \int dx = x + C_2$$

Please turn over...

$$y(x) = C_1 \cos x + C_2 \sin x + \cos x \ln|\cos x| + x \sin x.$$

3. Solve by method of undetermined coefficients:

$$y^{(4)} + 2y'' + y = (x-1)^2.$$

Homogeneous: $m^4 + 2m^2 + 1 = 0$

$(m^2+1)^2 = 0$

$m^2 = -1$

$m = \pm i$ (multiplicity 2)

$y_h(x) = c_1 \cos x + c_2 \sin x + c_3 x \cos x + c_4 x \sin x.$

Particular solution:

$$\begin{array}{r|l} 1 & y_p = Ax^2 + Bx + C \\ 2 & y_p'' = 2A \\ 1 & y_p^{(4)} = 0 \\ \hline + & Ax^2 + Bx + C + 4A = x^2 - 2x + 1 \end{array}$$

$\Rightarrow A = 1,\ B = -2,\ C + 4A = 1$

$\Rightarrow C = 1 - 4A \Rightarrow C = -3.$

$y_p(x) = x^2 - 2x - 3$

General solution:

$y(x) = x^2 - 2x - 3 + c_1 \cos x + c_2 \sin x + c_3 x \cos x + c_4 x \sin x.$