Wed, Jul 30, 2014 MATH 280, Quiz 4 Prof. V. Panferov

Name (print): Solutions.

Each problem is worth 2 points. Show all your work.

1. Verify that the given functions form a fundamental set of solutions of the given equation on $(-\infty, \infty)$:

$$y'' - y' - 12y = 0, \quad y_1 = e^{-3x}, \quad y_2 = e^{+4x}.$$

$$\begin{array}{r|l} -12\cdot & y_1 = e^{-3x} \\ -1\cdot & y_1' = -3e^{-3x} \\ 1\cdot & y_1'' = 9e^{-3x} \end{array}$$

$+$

$$y_1'' - y_1' - 12y_1 = (-12+3+9)e^{-3x} = 0$$

$$\begin{array}{r|l} -12 & y_2 = e^{4x} \\ -1 & y_2' = 4e^{4x} \\ 1 & y_2'' = 16e^{4x} \end{array}$$

$+$

$$y_2'' - y_2' - 12y_2 = (-12-4+16)e^{4x} = 0.$$

$$W(x) = \begin{vmatrix} e^{-3x} & e^{4x} \\ -3e^{-3x} & 4e^{4x} \end{vmatrix} = (4+3)e^{x} = 7e^{x} \neq 0$$

$\Rightarrow \{y_1, y_2\}$ – a fundamental set.

2. (a) Find the general solution of the equation $y'' - 10y' + 25y = 0$. (b) Find the particular solution that satisfies the initial conditions $y(0) = 0$, $y'(0) = 1$.

$$y'' - 10y' + 25y = 0$$

$$m^2 - 10m + 25 = 0$$

$$(m-5)^2 = 0$$

$m = 5$ (root of mult. 2)

$$y_1 = e^{5x}, \quad y_2 = xe^{5x}$$

$y(x) = c_1 e^{5x} + c_2 x e^{5x}$ – general solution

$$y(0) = c_1 = 0$$

$$y'(x) = 5\underbrace{c_1}_{=0} e^{5x} + c_2(5x+1)e^{5x}$$

$$y'(0) = c_2 = 1 \quad \Rightarrow y_p(x) = xe^{5x}.$$

Please turn over...

3. The indicated function y_1 is a solution of the given differential equation:

$$x^2y'' - 7xy' + 16y = 0, \qquad y_1(x) = x^4.$$

Use the process of <u>reduction of order</u> to find a second solution $y_2(x)$. Show all steps.

$$\begin{array}{r|l} 16 & y_2 = uy_1 \\ -7x & y_2' = u'y_1 + uy_1' \\ x^2 & y_2'' = u''y_1 + 2u'y_1' + uy_1'' \\ + & \end{array}$$

cancels

$$x^2y'' - 7xy' + 16y = x^6u'' + 8x^5u' - 7x^5u' = 0$$

$$x^6u'' + x^5u' = 0$$

$$xu'' + u = 0$$

$$v = u' \Rightarrow xv' + v = 0$$

$$(xv)' = 0$$

$$xv = c_1$$

$$v = \frac{c_1}{x}$$

$$u' = \frac{c_1}{x}$$

$$u = c_1 \ln|x| + c_2$$

$$y(x) = c_1 x^4 \ln|x| + c_2 x^4$$

$$y_2(x) = x^4 \ln|x|.$$

The end.