Tue, Jul 22, 2014 **MATH 280, Quiz 3** Prof. V. Panferov

Name (print): Solutions.

Each problem is worth 2 points. Show all your work.

1. Determine if the given differential equation is exact. If it is exact, solve it:

$$\underbrace{(x+y)^2}_{=M}\,dx + \underbrace{(2xy + x^2 - 1)}_{=N}\,dy = 0.$$

$M_y = 2(x+y)$; $N_x = 2y + 2x$; $M_y = N_x \Rightarrow$ exact.

$f(x,y) = \int (x+y)^2\,dx = \frac{1}{3}(x+y)^3 + g(y)$

$\frac{\partial f}{\partial y} = (x+y)^2 + g'(y) = x^2 + 2xy + y^2 + g'(y)$

$= N = x^2 + 2xy - 1$

$y^2 + g'(y) = -1$

$g'(y) = -y^2 - 1 \Rightarrow g(y) = -\frac{1}{3}y^3 - y + C$

Gen. solution

$\frac{1}{3}(x+y)^3 - \frac{1}{3}y^3 - y = C$

or $\frac{1}{3}x^3 + x^2y + xy^2 - y = C$

2. Solve the homogeneous equation: $(y^2 + yx)\,dx - x^2\,dy = 0.$

$\frac{y}{x} = u$, $y = ux$; $dy = u\,dx + x\,du$

$(u^2x^2 + \cancel{ux^2})\,dx - x^2(\cancel{u\,dx} + x\,du) = 0$

$u^2x^2\,dx - x^3\,du = 0$

$u^2\,dx = x\,du$

$\frac{du}{u^2} = \frac{dx}{x}$

$\int \frac{du}{u^2} = \int \frac{dx}{x} = \ln|x| + C$

$-\frac{1}{u} = \ln|x| + c$

$-\frac{x}{y} = \ln|x| + c$

$y = \frac{x}{C - \ln|x|}$

or

$y\ln|x| + x + Cy = 0$

Please turn over...

3. Solve the initial value problem for the Bernoulli equation:

$$xy' + y = y^{-2}, \quad y(1) = 2.$$

$n = -2$, $\quad u = y^{1-n} = y^3$; $\quad y = u^{\frac{1}{3}}$

$$y' = \frac{1}{3} u^{-\frac{2}{3}} u'$$

$$\frac{1}{3} x u^{-\frac{2}{3}} u' + u^{\frac{1}{3}} = u^{-\frac{2}{3}}$$

$$\frac{1}{3} x u' + u = 1$$

$$u' + \frac{3}{x} u = \frac{3}{x}$$

$$\mu(x) = e^{\int \frac{3}{x} dx} = e^{3 \ln x} = x^3 \quad (x > 0)$$

$$x^3 u' + 3x^2 u = 3x^2$$

$$(x^3 u)' = 3x^2$$

$$x^3 u = x^3 + C$$

$$u = 1 + Cx^{-3}$$

$$y^3 = 1 + Cx^{-3}$$

$$y = (1 + Cx^{-3})^{1/3}$$

$$y(1) = (1 + C)^{1/3} = 2$$

$$1 + C = 2^3 = 8$$

$$C = 7$$

$$y = (1 + 7x^{-3})^{1/3}.$$

The end.