Thu, Jul 17, 2014 **MATH 280, Quiz 2** Prof. V. Panferov

Name (print): Solutions.

Each problem is worth 2 points. Show all your work.

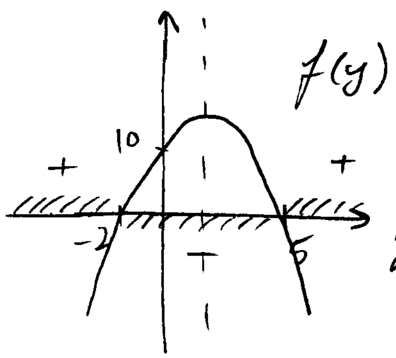
1. Find all critical points and sketch the "phase portrait" of the autonomous equation:

$$\frac{dy}{dx} = 10 + 3y - y^2.$$

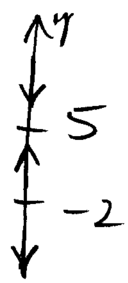
Sketch a graph showing the equilibrium solutions a few typical solution curves. Classify equilibrium solutions as asymptotically stable, unstable or semi-stable.

$$f(y) = 10 + 3y - y^2 = (5-y)(2+y)$$

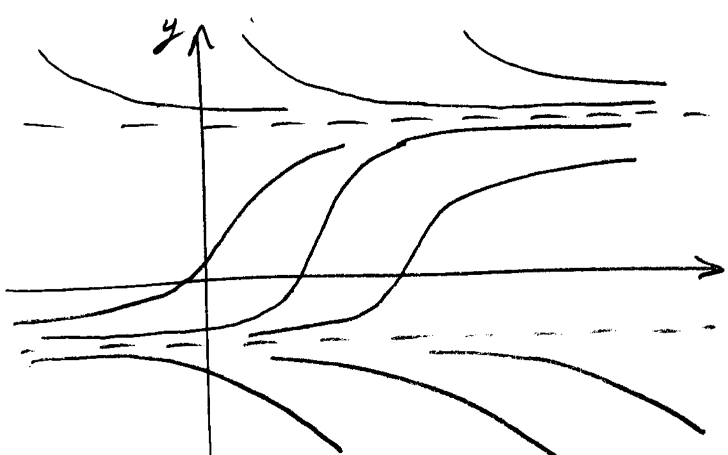
$$f(y) = 0 \iff \underbrace{y = -2 \text{ or } y = 5}_{\text{critical points.}}$$



$\Rightarrow$



"phase portrait"



$y = 5$ — asymp. stable as $x \to \infty$

$y = -2$ — unstable as $x \to \infty$

Solution curves ↗

Please turn over...

2. (a) Use the method of separation of variables to solve the differential equation:

$$\frac{dy}{dx} = x\sqrt{1-y^2}.$$

(b) Find all singular solutions that were "lost" during the separation of variables.

(a)

$$\frac{dy}{\sqrt{1-y^2}} = x\,dx$$

$$\int \frac{dy}{\sqrt{1-y^2}} = \int x\,dx = \frac{x^2}{2} + C$$

$$\arcsin y = \frac{x^2}{2} + C$$

$$\underline{y = \sin\left(\frac{x^2}{2} + C\right)}$$

(b) To obtain solutions above we divided by $\sqrt{1-y^2}$.

$$\sqrt{1-y^2} = 0 \iff 1-y^2 = 0 \iff y = \pm 1.$$

$\underline{y(x) = \pm 1}$ are constant, singular solutions

(verify: $y' = 0$ and $x\sqrt{1-y^2} = 0$.)

3. (a) Solve the equation $dy/dx = x/y$ using separation of variables

(b) Find explicit solutions $y = y(x)$ of equation in part (a) with initial data (i) $y(1) = 1$ and (ii) $y(2) = 1$. Indicate the intervals of existence.

(a) $y\,dy = x\,dx$

$$\int y\,dy = \int x\,dx = \frac{x^2}{2} + C_1$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C_1$$

$$\underline{y^2 - x^2 = C} \quad (= 2C_1).$$

– implicit solution

(b) (i) $y(1) = 1 \Rightarrow 1^2 - 1^2 = C \Rightarrow C = 0$

$y^2 = x^2 \Rightarrow \underline{y = x}$ since $y(1) > 0$.

Largest interval of existence is $\underline{(0, \infty)}$, since equation becomes undefined when $y = 0$, i.e. $x = 0$.

(ii) $y(2) = 1 \Rightarrow 1^2 - 2^2 = C \Rightarrow C = -3$.

$y^2 = x^2 - 3 \Rightarrow \underline{y = \sqrt{x^2 - 3}}$ since $y(2) > 0$.

Largest interval of existence is $\underline{(\sqrt{3}, \infty)}$.

Since $y(x) = \sqrt{x^2 - 3}$ is defined for $[\sqrt{3}, \infty)$ and derivative is not defined at $x = \sqrt{3}$.

The end.