Tue, Jul 15, 2014 MATH 280, Quiz 1 Prof. V. Panferov

Name (print): Solutions.

Each problem is worth 2 points. Show all your work.

1. Verify that the function $y = x + 4\sqrt{x+2}$ is a solution of the differential equation

$$(y-x)y' = y - x + 8.$$

Give an interval of definition of the solution $y(x)$ which contains $x = 0$.

$$y' = 1 + \frac{4}{2\sqrt{x+2}} = 1 + \frac{2}{\sqrt{x+2}}.$$

$$L.H.S. = (y-x)y' = 4\sqrt{x+2}\left(1 + \frac{2}{\sqrt{x+2}}\right) = 4\sqrt{x+2} + 8.$$

$$R.H.S = y - x + 8 = 4\sqrt{x+2} + 8 \longleftarrow = \checkmark$$

domain of the solution: $x+2 \geq 0$ or $x \geq -2$

the derivative is not defined at $x = -2$

Largest interval of existence of the solution: $I = (-2, \infty)$.

2. Find a linear second-order differential equation $F(x, y, y', y'') = 0$ for which $y = c_1x + c_2x^2$ is a two-parameter family of solutions. (Make sure that your equation is free of the arbitrary parameters c_1 and c_2.)

$$y = c_1 x + c_2 x^2 \quad | \cdot 1 \qquad\qquad y = c_1 x + c_2 x^2$$

$$y' = c_1 + 2c_2 x \quad | \cdot x \quad \Rightarrow \quad xy' = c_1 x + 2c_2 x^2$$

$$y'' = 2c_2 \quad | \cdot x^2 \qquad\qquad x^2 y'' = 2c_2 x^2$$

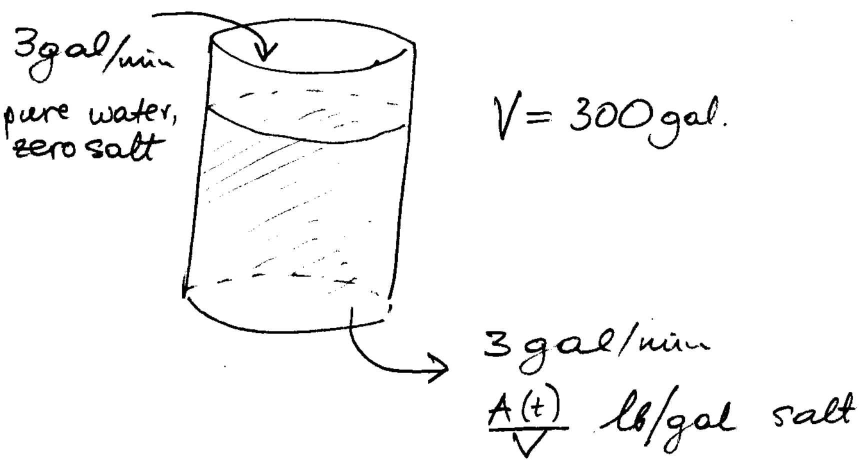
Find a combination for which all terms with c_1, c_2 cancel:

$$-\frac{1}{2}x^2y'' + xy' = c_1 x + c_2 x^2$$

$$-\frac{1}{2}x^2y'' + xy' - y = 0$$ — linear, 2^{nd} order diff. equation.

Please turn over...

3. Suppose that a large mixing tank initially holds 300 gallons of water in which 50 pounds of salt have been dissolved. Pure water is pumped into the tank at a rate of 3 gal/min, and when the solution is well-stirred, it is then pumped out at the same rate. Determine a differential equation for the amount of salt $A(t)$ in the tank at time t. (Show how the equation is derived.) What is $A(0)$?



$V = 300\,\text{gal.}$

$A(t)$ - amount of salt, in lb.

Balance of salt in the tank, from t to $t+\Delta t$.

$$A(t+\Delta t) = A(t) + 0 \cdot 3 \left[\frac{lb}{min}\right]\Delta t - 3 \cdot \frac{A(t)}{V}\left[\frac{lb}{min}\right]\Delta t$$

$$A(t+\Delta t) = A(t) - \frac{3}{300} A(t)\,\Delta t$$

$$\frac{A(t+\Delta t) - A(t)}{\Delta t} = -\frac{1}{100} A(t)$$

Let $\Delta t \to 0$!

$$\underline{\frac{dA}{dt} = -\frac{1}{100} A} \quad \text{- diff. equation.}$$

Initial condition: $\underline{A(0) = 50 \;[lb.]}$