Thu, Jun 25, 2013 **MATH 250, Quiz 7** Prof. V. Panferov

Name (print): Solutions.

There are two problems worth 2 points and one problem worth 4 points. Show all your work.

1. (2 points) Find the directional derivative of f at the given point in the direction indicated by the angle θ:

$$f(x,y) = ye^{-x}, \quad P(0,4), \quad \theta = 2\pi/3.$$

$$f_x = -ye^{-x}, \quad f_y = e^{-x}$$

$$f_x(0,4) = -4, \quad f_y(0,4) = 1$$

$$D_{\vec{u}} f(0,4) = f_x(0,4)\cos\theta + f_y(0,4)\sin\theta$$

$$= -4\left(-\frac{1}{2}\right) + 1 \cdot \frac{\sqrt{3}}{2} = 2 + \frac{\sqrt{3}}{2}$$

2. (2 points) Find an equation of the tangent plane to the surface $xyz^2 = 6$ at the point $P(3,2,1)$.

$$F(x,y,z) = xyz^2$$

$$F_x = yz^2, \quad F_y = xz^2, \quad F_z = 2xyz$$

when $x = 3, y = 2, z = 1$:

$$F_x = 2, \quad F_y = 3, \quad F_z = 12$$

Tangent plane:

$$2(x-3) + 3(y-2) + 12(z-1) = 0$$

$$2x + 3y + 12z = 24$$

Please turn over...

3. (4 points) Find all critical points and classify them as local maxima, local minima or saddle points:

$$f(x,y) = x^3 - 12xy + 8y^3.$$

$$\vec{\nabla} f = \left(3x^2 - 12y,\ -12x + 24y^2\right)$$

Crit. points: $\left.\begin{array}{l} 3x^2 - 12y = 0 \\ -12x + 24y^2 = 0 \end{array}\right\} \Rightarrow \begin{array}{l} x^2 = 4y \\ x = 2y^2 \end{array}$

$\Rightarrow \quad 4y^4 = 4y$

$\Rightarrow y(y^3 - 1) = 0$

$\Rightarrow y = 0$ or $y = 1$ $\Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$ or $\begin{cases} x = 2 \\ y = 1 \end{cases}$

Second derivatives:

$$D^2 f = \begin{pmatrix} 6x & -12 \\ -12 & 48y \end{pmatrix}$$

$(x,y) = (0,0)$

$$D^2 f = \begin{pmatrix} 0 & -12 \\ -12 & 0 \end{pmatrix}$$

$\Delta = -144 < 0$

$\Rightarrow$ saddle point at $(0,0)$

$(x,y) = (2,1)$

$$D^2 f = \begin{pmatrix} 12 & -12 \\ -12 & 48 \end{pmatrix}$$

$= 12 \cdot 48 - 12 \cdot 12 > 0$

$f_{xx} = 12 > 0$

$\Rightarrow$ local minimum at $(2,1)$.