Thu, Jun 20, 2013 **MATH 250, Quiz 6** Prof. V. Panferov

Name (print): Solutions.

Each problem is worth 2 points. Show all your work.

1. Find an equation of the tangent plane to the graph of the function

$$z = x\sin(x+y), \quad \text{at the point} \quad P(-1,1,0).$$

Express your answer in the form $ax+by+cz=d$.

$$f(x,y) = x\sin(x+y)$$

$$f_x = \sin(x+y) + x\cos(x+y) \Rightarrow f_x(-1,1) = -1.$$

$$f_y = x\cos(x+y) \Rightarrow f_y(-1,1) = -1.$$

Tangent plane:

$$z - z_0 = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

$$z - 0 = (-1)(x+1) + (-1)\cdot(y-1)$$

$$z = -x-y$$

$$x+y+z = 0.$$

2. Use the chain rule to find the partial derivative $\partial z/\partial t$:

$$z = x^2y^3, \quad x = s\cos t, \quad y = s\sin t.$$

$$\frac{\partial z}{\partial x} = 2xy^3 \; ; \quad \frac{\partial z}{\partial y} = 3x^2y^2$$

$$\frac{\partial x}{\partial t} = -s\sin t \; ; \quad \frac{\partial y}{\partial t} = s\cos t$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

$$= 2xy^3(-s\cdot\sin t) + 3x^2y^2(s\cdot\cos t)$$

$$= -2s^5\cos t\sin^4 t + 3s^5\cos^3 t\sin^2 t.$$

Please turn over...

3. Find the linearization $L(x, y)$ of the function at the point (x_0, y_0):

$$f(x, y) = \frac{x}{x+y}, \quad (x_0, y_0) = (2, 1).$$

Explain why the function is differentiable at that point.

$$f_x = \frac{(x+y) - x}{(x+y)^2} = \frac{y}{(x+y)^2} \ ; \quad f_y = -\frac{x}{(x+y)^2}$$

$$f_x(2,1) = \frac{1}{9} \ ; \quad f_y(2,1) = -\frac{2}{9}$$

$$L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

$$= \frac{2}{3} + \frac{1}{9}(x-2) - \frac{2}{9}(y-1)$$

$$= \frac{2}{3} + \frac{1}{9}x - \frac{2}{9}y.$$

f_x, f_y are continuous at $(2,1)$

$\Rightarrow f(x,y)$ is differentiable at $(2,1)$, by the Fundamental Lemma on Differentiation

4. The length and width of a rectangle are measured as 30 cm and 24 cm, respectively, with an error of measurement of at most 0.1 cm in each. Use differentials to estimate the maximum error in the calculated area of the rectangle.

$$f(x,y) = xy \ ; \quad (x_0, y_0) = (30, 24)$$

$$|\Delta x|, |\Delta y| \leq 0.1$$

$$\Delta f = \underbrace{f_x(x_0,y_0)\,\Delta x + f_y(x_0,y_0)\,\Delta y}_{\text{differential } df} + \text{remainder}$$

$$f_x = y \Rightarrow f_x(x_0,y_0) = 24$$

$$f_y = x \Rightarrow f_y(x_0,y_0) = 30.$$

$$df = 24\,\Delta x + 30\,\Delta y$$

$$|df| \leq 24 \cdot 0.1 + 30 \cdot 0.1 = 5.4 \text{ cm}^2$$

$$|\Delta f| \lesssim 5.4 \text{ cm}^2.$$