Tue, Jun 18, 2013 **MATH 250, Quiz 5** Prof. V. Panferov

Name (print): Solutions.

Each problem is worth 2 points. Show all your work.

1. Find the partial derivatives $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial z}$:

$$w = xz - 5x^2y^3z^4.$$

$$\frac{\partial w}{\partial x} = z - 10xy^3z^4$$

$$\frac{\partial w}{\partial z} = x - 20x^2y^3z^3.$$

2. Find and sketch the domain of the function:

$$f(x,y) = \sqrt{1-x^2} - \sqrt{1-y^2}.$$

$$D = \{(x,y) : 1-x^2 \geq 0,\ 1-y^2 \geq 0\} = [-1,1]^2$$

$1-x^2 \geq 0$

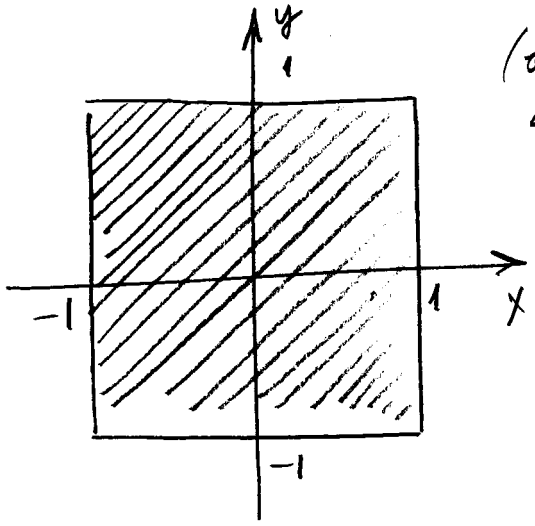
$\Leftrightarrow\ x^2 \leq 1$

$\Leftrightarrow\ -1 \leq x \leq 1$

likewise,

$1-y^2 \geq 0$

$\Leftrightarrow\ -1 \leq y \leq 1$



(boundary of the square included)

Please turn over...

3. Determine whether the function $f(x,y)$ is continuous at $(0,0)$:

$$f(x,y) = \begin{cases} \dfrac{x^2y^3}{2x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$$

$$\frac{x^2y^3}{2x^2+y^2} = \left(\frac{xy}{\sqrt{2}}\right)^2 \cdot \frac{\sqrt{2}x \cdot y}{2x^2+y^2}$$

$$-\frac{1}{2\sqrt{2}}|x|y^2 \leq f(x,y) \leq \frac{1}{2\sqrt{2}}|x|y^2$$

$(x,y)\to(0,0)$: 0 — By Squeezing Principle — $(x,y)\to(0,0)$: 0

$\Rightarrow$ $f(x,y)$ is continuous at $(0,0)$.

4. The ideal gas law for fixed mass m reads $PV = mRT$ where P is the pressure, V is the volume occupied by the gas, T is the temperature, and R is the gas constant. Find the partial derivatives

$$\frac{\partial P}{\partial V} \text{ and } \frac{\partial V}{\partial T}.$$

$$P = \frac{mRT}{V} \Rightarrow \frac{\partial P}{\partial V} = -\frac{mRT}{V^2}$$

$$V = \frac{mRT}{P} \Rightarrow \frac{\partial V}{\partial T} = \frac{mR}{P}.$$