Tue, Jun 11, 2013 **MATH 250, Quiz 4** Prof. V. Panferov

Name (print): Solutions.

Each problem is worth 2 points. Show all your work.

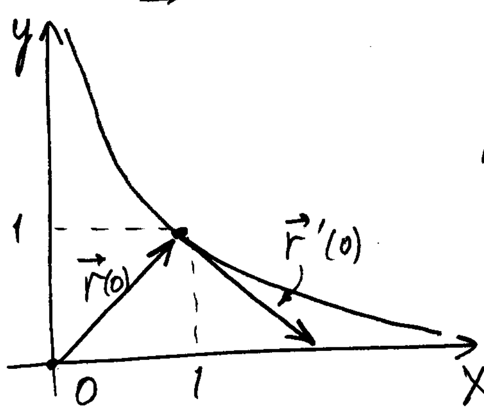
1. Sketch the curve with the given vector equation. Find $\vec{r}'(t)$. Sketch the vectors $\vec{r}(t)$ and $\vec{r}'(t)$ for the given value of t:

$$\vec{r}(t) = e^t\vec{i} + e^{-t}\vec{j}, \quad t = 0.$$

$\left.\begin{array}{l} x = e^t > 0 \\ y = e^{-t} > 0 \end{array}\right\} \Rightarrow$ the curve is in the first quadrant.

$$y = e^{-t} = \frac{1}{e^t} = \frac{1}{x}$$

$\Rightarrow$ the curve is part of the hyperbola $y = \frac{1}{x}$.



$$\vec{r}'(t) = e^t\vec{i} - e^{-t}\vec{j}$$

$$\vec{r}(0) = \vec{i} + \vec{j} = (1, 1)$$

$$\vec{r}'(0) = \vec{i} - \vec{j} = (1, -1)$$

2. Find the derivative of the vector function $\vec{r}(t) = e^{t^2}\vec{i} + \vec{j} + \ln(1+3t)\,\vec{k}$.

$$\vec{r}'(t) = 2te^{t^2}\vec{i} + 0\cdot\vec{j} + \frac{3}{1+3t}\vec{k}$$

$$= \left(2te^{t^2}, 0, \frac{3}{1+3t}\right).$$

3. Find the integral:

$$\int \overbrace{\left(\sec^2 t\,\vec{i} + 3\sin^2 t\,\cos t\,\vec{j} + t^2 \ln t\,\vec{k}\right)}^{=\vec{r}(t)} dt$$

$$\int \sec^2 t\, dt = \tan t + C_1$$

$$\int 3\sin^2 t \cos t\, dt = \int 3u^2 du \Big|_{u=\cos t} = (\cos t)^3 + C_2$$

$$\int t^2 \ln t\, dt = \int \left(\frac{1}{3}t^3\right)' \ln t\, dt$$

$$= \frac{1}{3}t^3 \ln t - \int \frac{1}{3}t^3 \frac{1}{t}\, dt$$

$$= \frac{1}{3}t^3 \ln t - \frac{1}{9}t^3 + C_3$$

$$\int \vec{r}(t)\,dt = \tan t\,\vec{i} + \cos^3 t\,\vec{j} + \frac{t^3}{9}(3\ln t - 1)\vec{k} + \vec{C}$$

where $\vec{C} = C_1\vec{i} + C_2\vec{j} + C_3\vec{k}$.

4. Find the length of the curve:

$$\vec{r}(t) = \vec{i} + t^2\vec{j} + t^3\vec{k}, \quad 0 \le t \le 1.$$

$$\vec{r}\,'(t) = 2t\vec{j} + 3t^2\vec{k}$$

$$|\vec{r}\,'(t)| = \sqrt{4t^2 + 9t^4}$$

$$\ell(C) = \int_0^1 \sqrt{4t^2 + 9t^4}\, dt = \int_0^1 2t\sqrt{1 + \frac{9}{4}t^2}\, dt$$

$$= \int_0^1 \sqrt{1 + \frac{9}{4}u}\, du = \left[\frac{4}{9}\frac{2}{3}\left(1 + \frac{9}{4}u\right)^{3/2}\right]_{u=0}^{u=1}$$

$(u = t^2)$

$$= \frac{8}{27}\left(\left(1 + \frac{9}{4}\right)^{3/2} - 1\right) = \frac{8}{27}\left(\frac{13\sqrt{13}}{8} - 1\right)$$

$$= \frac{13\sqrt{13} - 8}{27} \approx 1.4397$$