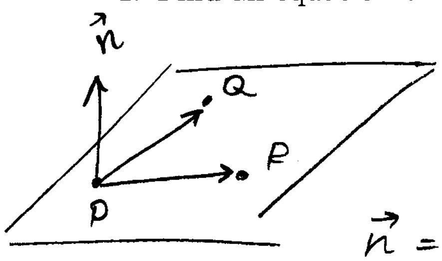
Mon Jun 10, 2013 **MATH 250, Quiz 3** Prof. V. Panferov

Name (print): Solutions.

Each problem is worth 2 points. Show all your work.

1. Find an equation of the plane through the points $P(0,1,1)$, $Q(1,0,1)$ and $R(1,1,0)$.




$$\vec{PQ} = (1,-1,0) \qquad \vec{PR} = (1,0,-1)$$

$$\vec{n} = \vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = \vec{i} + \vec{j} + \vec{k}$$

Plane through $P(0,1,1)$ with normal vector $\vec{n}$:

$$(x-0) + (y-1) + (z-1) = 0$$

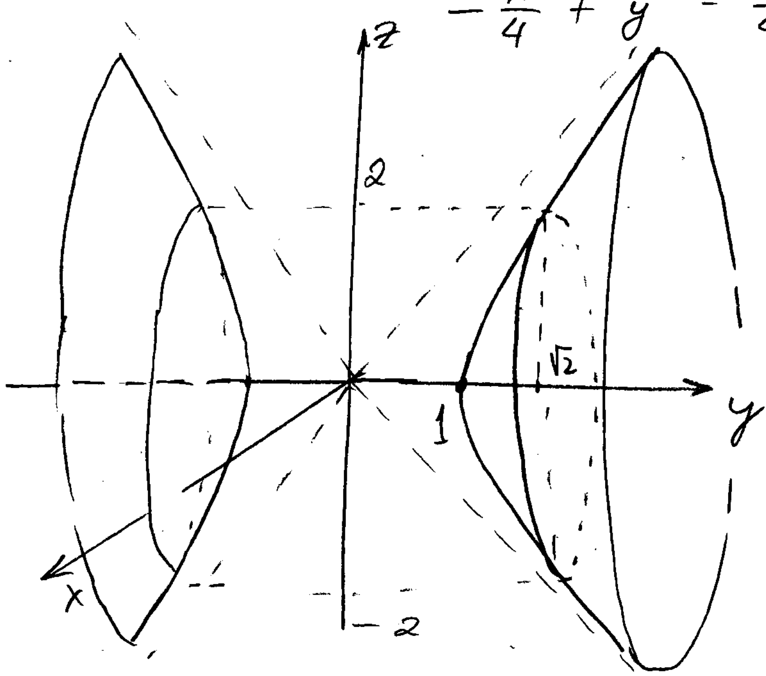
or

$$x + y + z - 2 = 0.$$

2. Sketch the surface given by the equation

$$-x^2 + 4y^2 - z^2 = 4.$$

$$-\frac{x^2}{4} + y^2 - \frac{z^2}{4} = 1$$




$x = 0$:

$y^2 - \left(\frac{z}{2}\right)^2 = 1$

- hyperbola with asymptotes $z = \pm 2y$

hyperboloid in 2 sheets

axis of symmetry - the y-axis.

*Please turn over...*

3. Find the limit:

$$\lim_{t\to 0}\overbrace{\left(e^{-t}\vec{\imath}+\frac{t^2}{\sin^2 t}\vec{\jmath}+\cos 2t\,\vec{k}\right)}^{\vec{r}\,(t)}.$$

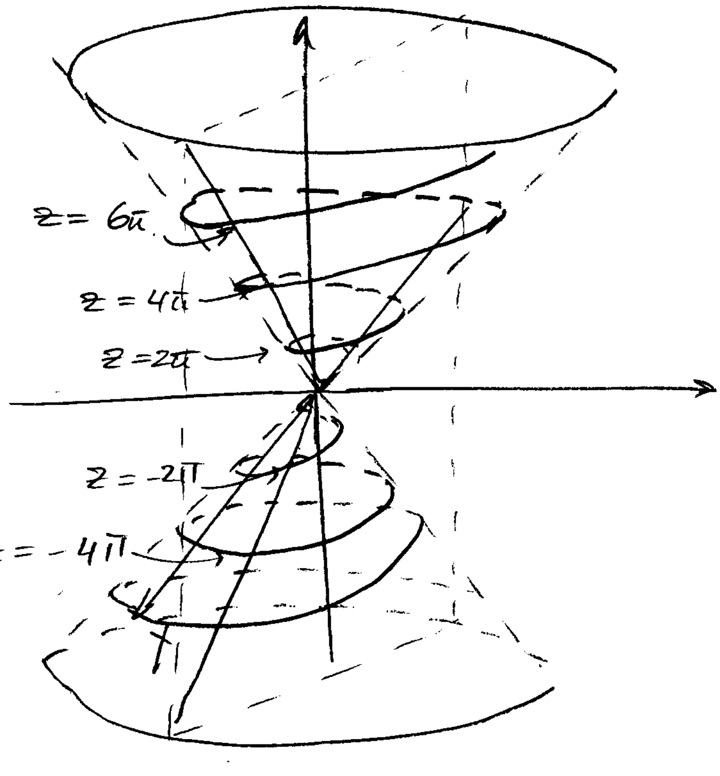
$$\lim_{t\to 0} e^{-t} = 1; \quad \lim_{t\to 0}\frac{t^2}{\sin^2 t} = \Big(\underbrace{\lim_{t\to 0}\frac{t}{\sin t}}_{=1 \text{ by L'Hopital's rule}}\Big)^2 = 1$$

$$\lim_{t\to 0}\cos 2t = 1$$

$$\lim_{t\to 0}\vec{r}\,(t) = (1, 1, 1).$$

4. Show that the curve with parametric equations $x = t\cos t$, $y = t\sin t$, $z = t$ lies on the cone $z^2 = x^2 + y^2$ and use this fact to sketch the curve.

$$z^2 = t^2 = t^2\cos^2 t + t^2\sin^2 t = x^2 + y^2.$$




Spiral, expanding from the origin, on the circular cone

$$x^2 + y^2 = z^2.$$