

Name (print): Solutions.

Each problem is worth 2 points. Show all your work.

1. Determine whether the points lie on straight line:

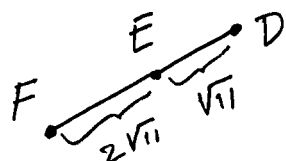
$$D(0, -5, 5), \quad E(1, -2, 4), \quad F(3, 4, 2).$$

One way:

$$|DE| = \sqrt{1^2 + 3^2 + 9^2} = \sqrt{11}$$

$$|DF| = \sqrt{3^2 + 9^2 + 3^2} = \sqrt{99} = 3\sqrt{11}$$

$$|EF| = \sqrt{2^2 + 6^2 + 2^2} = \sqrt{44} = 2\sqrt{11}$$



$\Rightarrow |DF| = |EF| + |DE| \Rightarrow$ the three points are on the same line.

Another way: $\vec{DE} = (1, 3, -1)$; $\vec{DF} = (3, 9, -3)$

$\vec{DF} = 3\vec{DE} \Rightarrow$ vectors are parallel
 $\Rightarrow$ points are on the same line.

2. Find an equation of the sphere that passes through the point $P(4, 3, -1)$ and has center $Q(3, 8, 1)$.

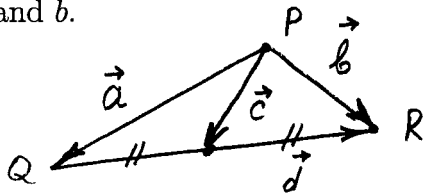
$$|PQ| = \sqrt{1^2 + 5^2 + 2^2} = \sqrt{30} \quad \text{— radius of the sphere.}$$

Sphere has center at $Q \Rightarrow$

$$(x-3)^2 + (y-8)^2 + (z-1)^2 = 30$$

$$\left(\text{or } x^2 + y^2 + z^2 - 6x - 16y - 2z + 44 = 0 \right)$$

3. In the figure, the tip of $\vec{c}$ and the tail of $\vec{d}$ are both the midpoint of QR . Express the $\vec{c}$ and $\vec{d}$ in terms of $\vec{a}$ and $\vec{b}$.



$$\vec{c} = \frac{\vec{a} + \vec{b}}{2} \quad - \text{parallelogram rule}$$

$$\vec{d} = \frac{\vec{b} - \vec{a}}{2}, \text{ Since } \vec{QR} = \vec{b} - \vec{a}.$$

4. Find $|\vec{a} - \vec{b}|$:

$$\vec{a} = \vec{i} + 2\vec{j} - 3\vec{k}, \quad \vec{b} = -2\vec{i} - \vec{j} + 5\vec{k}.$$

$$\vec{a} - \vec{b} = 3\vec{i} + 3\vec{j} - 8\vec{k}$$

$$|\vec{a} - \vec{b}| = \sqrt{3^2 + 3^2 + 8^2} = \sqrt{82}$$