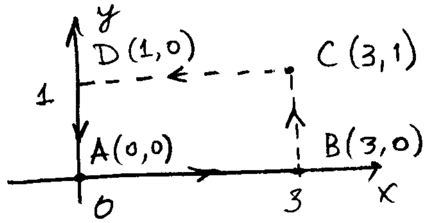
MATH 250

SECTION 16.4 - SELECTED EVEN PROBLEMS.

#2. (16.4)

$$\oint_C xy\,dx + x^2\,dy = \int_{C_1} + \int_{C_2} + \int_{C_3} + \int_{C_4}$$



Direct method.

$$(0,0) \xrightarrow{C_1} (3,0) \xrightarrow{C_2} (3,1) \xrightarrow{C_3} (1,0) \xrightarrow{C_4} (0,0)$$

$$= \underbrace{\int_0^3 x\cdot 0\,dx + x^2\cdot 0}_{\substack{C_1:\ dy=0,\ y=0 \\ 0\le x\le 3}}$$

$$+ \underbrace{\int_0^1 3y\cdot 0 + 3^2 dy}_{\substack{C_2:\ x=3,\ dx=0 \\ 0\le y\le 1}} + \underbrace{\int_3^0 x\cdot 1\cdot dx + x^2\cdot 0}_{\substack{C_3:\ y=1,\ dy=0 \\ 0\le x\le 3 \text{ (in reverse direction.)}}}$$

$$+ \underbrace{\int_1^0 0\cdot y\cdot 0 + 0\cdot dy}_{\substack{C_4:\ x=0,\ dx=0 \\ 0\le y\le 1 \text{ (in reverse direction)}}} =$$

$$= 0 + 9 - \frac{9}{2} + 0 = \frac{9}{2}.$$

Green's theorem:

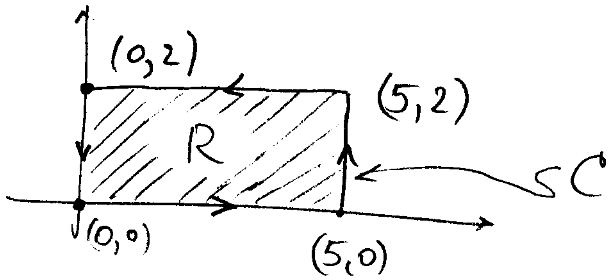
$$\oint_C xy\,dx + x^2\,dy = \iint_R \left(\frac{\partial}{\partial x} x^2 - \frac{\partial}{\partial y} xy\right) dx\,dy$$

$$= \iint_R (2x - x)\,dx\,dy = \iint_R x\,dx\,dy$$

$$= \int_0^1 \int_0^3 x\,dx\,dy = \int_0^1 \frac{9}{2}\,dy = \frac{9}{2}.$$

#6. (16.4)

$$\oint_C \cos y\, dx + x^2 \sin y\, dy =$$



$$= \iint_R \frac{\partial}{\partial x} x^2 \sin y - \frac{\partial}{\partial y} \cos y \, dx dy$$

$$= \iint_R 2x \sin y + \sin y \, dx dy$$

$$= \int_0^2 \int_0^5 (2x+1) \sin y \, dy\, dx$$

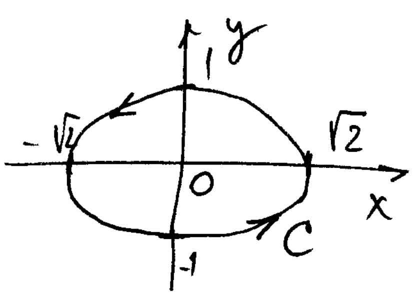
$$= \int_0^2 \sin y \, dy \int_0^5 (2x+1)\, dx = \Big[-\cos y\Big]_{y=0}^{y=2} \Big[x^2+x\Big]_{x=0}^{x=}$$

$$= (1-\cos 2)\cdot(25+5) = 30(1-\cos 2)$$

$$= 60 \sin^2(1) \approx 42.4844.$$

#8.

$$\oint_C y^4 dx + 2xy^3 dy = \iint_R \frac{\partial}{\partial x} 2xy^3 - \frac{\partial}{\partial y} y^4 \, dx dy$$



C: $x^2 + 2y^2 = 2$

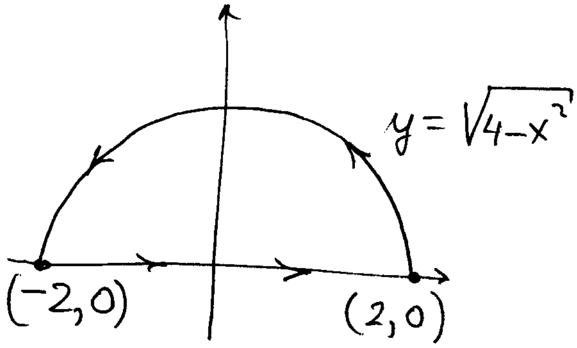
$$\frac{x^2}{2} + y^2 = 1$$

$$= \iint_R 2y^3 - 4y^3 \, dx dy = \iint_R -2y^3 dx dy$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \underbrace{\int_{-\sqrt{1-\frac{x^2}{2}}}^{\sqrt{1-\frac{x^2}{2}}} -2y^3\, dy}_{=0} \, dx = 0.$$

since y^3 is an odd function.

18.



$\vec{F}(x,y) = (x,\ x^3+3xy^2)$

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 3x^2+3y^2-0$$

$$\neq 0$$

So $\vec{F}$ is not conservative.

Compute

$$\oint_C \vec{F}\cdot d\vec{r} = \oint_C P\,dx + Q\,dy = \iint_R \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\,dx\,dy.$$

$$= \iint_R 3(x^2+y^2)\,dx\,dy = \int_0^{\pi}\int_0^2 3r^2\cdot r\,dr\,d\theta$$

$$= \int_0^{\pi} d\theta \int_0^2 3r^3\,dr = \pi\left[\frac{3r^4}{4}\right]_0^2$$

$$= \pi\cdot\frac{3\cdot 16}{4} = 12\pi.$$