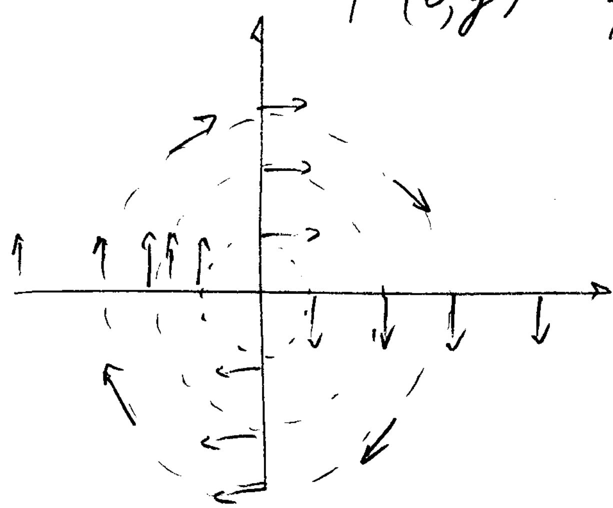
SECTION 16.1 – SELECTED EVEN PROBLEMS

#6. $\vec{F}(x,y) = \dfrac{y\vec{i} - x\vec{j}}{\sqrt{x^2+y^2}}$

Then $\left|\vec{F}(x,y)\right| = \dfrac{y^2 + x^2}{\left(\sqrt{x^2+y^2}\right)^2} = 1 \quad \left((x,y) \neq (0,0)\right)$

$\vec{F}(x,y) \cdot (x,y) = 0$

$\vec{F}(0,y) = \dfrac{y}{|y|}\vec{i}\,; \quad \vec{F}(x,0) = -\dfrac{x}{|x|}\vec{j}$



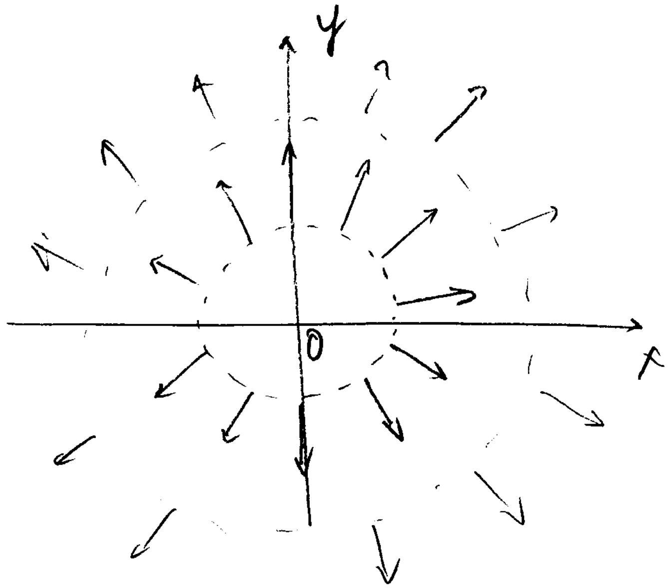
The vector field $\vec{F}(x,y)$ is tangent to the circles $x^2+y^2=r^2$.

#24. $f(x,y,z) = x \ln(y-2z)$

$\vec{\nabla} f = \left(\ln(y-2z),\ \dfrac{x}{y-2z},\ \dfrac{-2x}{y-2z}\right)$

#26. $f(x,y) = \sqrt{x^2+y^2}$

$\vec{\nabla} f = \left(\dfrac{x}{\sqrt{x^2+y^2}},\ \dfrac{y}{\sqrt{x^2+y^2}}\right)$



$|\vec{\nabla} f| = 1,$

$\vec{\nabla} f \parallel (x, y)$

The vector field is normal to each circle

$x^2 + y^2 = r^2.$

#29: $\underline{\text{III}}$ #30. $\underline{\text{IV}}$ # 31. $\underline{\text{II}}$ # 32: I

#29: $\vec{\nabla} f = (2x, 2y) \Rightarrow \vec{\nabla} f(\vec{r}) \parallel \vec{r}$

in fact $\vec{\nabla} f(\vec{r}) = 2\vec{r}$.

#30. $\vec{\nabla} f = (2x, x) \Rightarrow$ the vector field is independent of y

(all horizontal rows are the same)

#31. $\vec{\nabla} f = (2(x+y), 2(x+y))$

$\Rightarrow \vec{\nabla} f \parallel (1, 1)$ and $\vec{\nabla} f$ vanishes for $x = -y$.

#32. $\vec{\nabla} f = \cos\sqrt{x^2+y^2} \left(\frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}} \right)$

$\vec{\nabla} f(\vec{r}) \parallel \vec{r}$; $|\vec{\nabla} f|$ follows the $\cos r$ function.

(goes in radial waves.)