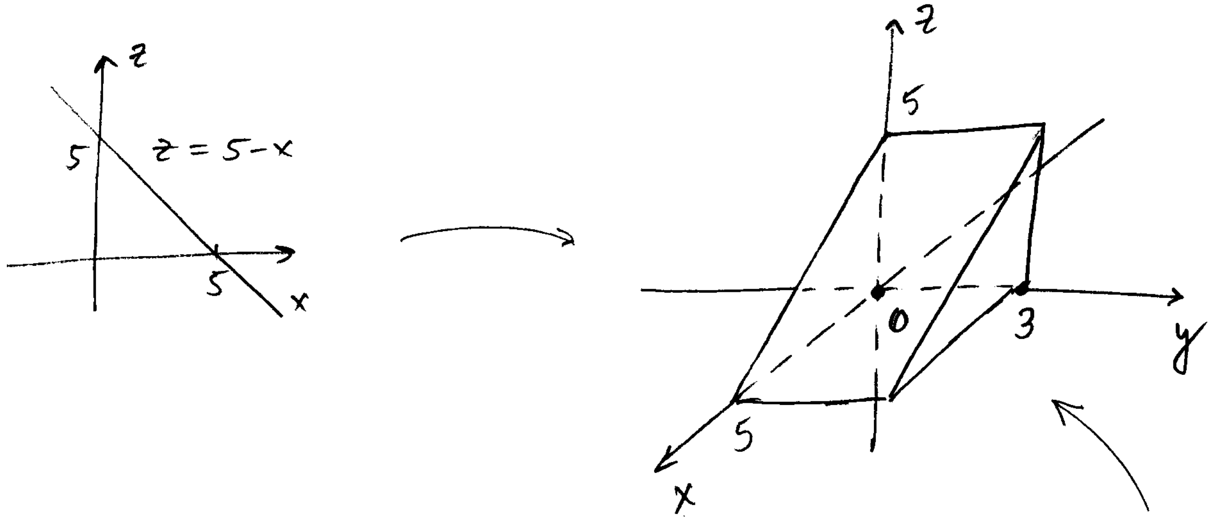
#12.

$$\iint_R (5-x)\,dx\,dy; \quad R = \{(x,y):\ 0 \le x \le 5,\ 0 \le y \le 3\}.$$

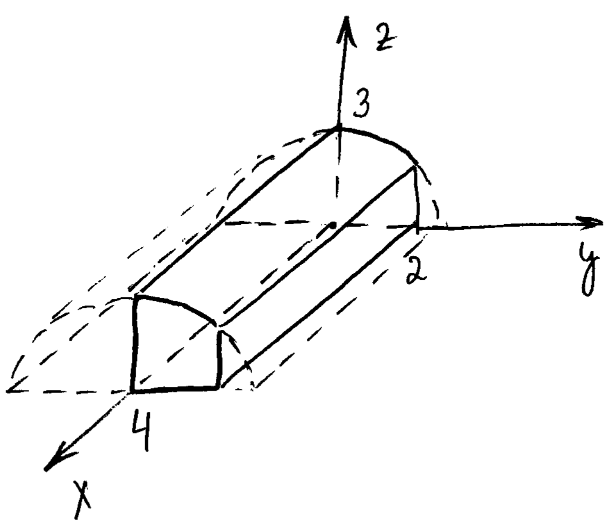


The solid is a wedge-shaped figure shown above. The volume of the wedge is $\frac{1}{2}$ of the volume of the rectangular box with dimensions

$$5 \times 3 \times 5 \implies V = \frac{1}{2}\cdot 5 \cdot 3 \cdot 5 = \frac{75}{2}.$$

#14.
(15.1)

$$\iint_R \sqrt{9-y^2}\,dx\,dy, \quad R = [0,4]\times[0,2].$$



The upper boundary of the three-dimensional region is the surface of the cylinder

$$z^2 = 9 - y^2,$$

which the x-axis as its axis of symmetry.

(The volume of the solid figure can be found using the formula for the volume of a cylinder,

$$V = Ah \quad , \text{ where } h=4 \text{ is the height,}$$

and $A = \int_0^2 \sqrt{9-y^2}\,dy$ is the area of the base.

This can be found with a little effort: (and using trig. substitutions:)

$$\int \sqrt{9-y^2}\,dy = \int 3\cos\theta \cdot 3\cos\theta\,d\theta = 9\int \cos^2\theta\,d\theta$$

$y = 3\sin\theta$
$dy = 3\cos\theta\,d\theta$

$$= \frac{9}{2}\int 1-\cos(2\theta)\,d\theta$$

$$= \frac{9}{2}\theta - \frac{9}{4}\sin(2\theta) + C = \frac{9}{2}\theta - \frac{9}{2}\sin\theta\cos\theta + C$$

$$= \frac{9}{2}\arcsin\frac{y}{3} - \frac{y}{2}\sqrt{9-y^2} + C.$$

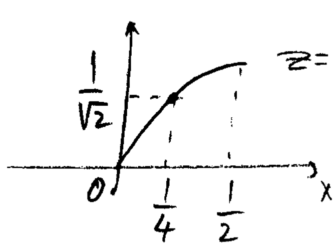
$$\Rightarrow A = \frac{9}{2}\arcsin\frac{2}{3} + \sqrt{5} \Rightarrow V = 18\arcsin\frac{2}{3} + 4\sqrt{5}.$$

\# 18. (15.1) First,

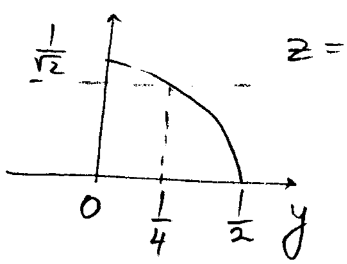
$$0 \le \sin \pi x \cos \pi y \le \frac{1}{2}$$

for $0 \le x \le \frac{1}{4}$, $\frac{1}{4} \le y \le \frac{1}{2}$.

Indeed based on the graphs:



$$0 \le \sin \pi x \le \frac{1}{\sqrt{2}}, \quad 0 \le x \le \frac{1}{4}.$$



$$0 \le \cos \pi y \le \frac{1}{\sqrt{2}}, \quad \frac{1}{4} \le y \le \frac{1}{2}$$

$\Rightarrow$ the product $\sin \pi x \cos \pi y$ is nonnegative and satisfies $\sin \pi x \cos \pi y \le \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$.

Since $\iint_R f \, dx\,dy \le \iint_R g \, dx\,dy$ if $f \le g$

(use Riemann sum definition, or the geometric definition as volume)

we obtain

$$\iint_R 0 \, dx\,dy \le \iint_R \sin \pi x \cos \pi y \, dx\,dy \le \iint_R \tfrac{1}{2} \, dx\,dy$$

$$= \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{32}.$$