MATH 250

SECTION 14.5 – SELECTED EVEN PROBLEMS

#4 (14.5)

$$z = \tan^{-1}\left(\frac{y}{x}\right), \quad x = e^t, \quad y = 1-e^{-t}$$

$$z_x = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \left(-\frac{y}{x^2}\right) = \frac{-y}{x^2+y^2}$$

$$z_y = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x} = \frac{x}{x^2+y^2}$$

$$x_t = e^t, \quad y_t = e^{-t}$$

$$z_t = z_x x_t + z_y y_t = \frac{-ye^t}{x^2+y^2} + \frac{xe^{-t}}{x^2+y^2}$$

$$= \frac{xe^{-t} - ye^t}{x^2+y^2}$$

#8. (14.5)

$$z = \arcsin(x-y), \quad x = s^2+t^2, \quad y = 1-2st$$

$$z_x = \frac{1}{\sqrt{1-(x-y)^2}}, \quad z_y = \frac{-1}{\sqrt{1-(x-y)^2}}$$

$$x_s = 2s \qquad y_s = -2t$$
$$x_t = 2t \qquad y_t = -2s$$

$$z_s = z_x x_s + z_y y_s = \frac{2s}{\sqrt{1-(x-y)^2}} + \frac{2t}{\sqrt{1-(x-y)^2}}$$

$$1-(x-y)^2 = 1-(s^2+t^2+2st-1)^2$$
$$= -(s+t)^4 + 2(s+t)^2 = (s+t)^2\left(2-(s+t)^2\right)$$

$$z_s = \frac{2(s+t)}{\sqrt{(s+t)^2(2-(s+t)^2)}} = \frac{2}{\sqrt{2-(s+t)^2}}.$$

#12.
(14.5)

$$z = \tan\frac{u}{v} \quad ; \quad u = 2s+3t, \quad v = 3s-2t$$

$$z_u = \frac{1}{1+\left(\frac{u}{v}\right)^2}\,\frac{1}{v} = \frac{v}{u^2+v^2}$$

$$z_v = \frac{1}{1+\left(\frac{u}{v}\right)^2}\left(-\frac{u}{v^2}\right) = \frac{-u}{u^2+v^2}$$

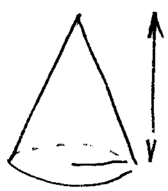
$$u_s = 2 \qquad v_s = 3$$

$$u_t = 3 \qquad v_t = -2$$

$$z_s = z_u u_s + z_v v_s = \frac{2v}{u^2+v^2} - \frac{3u}{u^2+v^2} = \frac{2v-3u}{u^2+v^2}$$

$$z_t = z_u u_t + z_v v_t = \frac{3v}{u^2+v^2} + \frac{2u}{u^2+v^2} = \frac{2u+3v}{u^2+v^2}$$

#38
(14.5)



$$V = \frac{1}{3}\pi r^2 h$$

$$r = r_0 + 1.8t \qquad (\text{in.})$$

$$h = h_0 - 2.5t \qquad (\text{in.})$$

Find $\frac{dV}{dt}$ when $r = 120$, $h = 140$

$$\frac{dV}{dt} = \frac{\partial V}{\partial r}\frac{dr}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt}$$

$$= \frac{2}{3}\pi r h \cdot 1.8 + \frac{1}{3}\pi r^2 \cdot (-2.5)$$

$$= \frac{1}{3}\pi r^2\left(3.6\,\frac{h}{r} - 2.5\right)$$

$$= \frac{1}{3}\pi \cdot 120^2\left(3.6\,\frac{140}{120} - 2.5\right)$$

$$= \pi \cdot 3 \cdot 40^2\left(3 \cdot 1.4 - 2.5\right)$$

$$= 4800\pi \cdot 1.7 \approx 25{,}635\ \frac{\text{in}^3}{s}.$$

$$\approx 14.84\ \frac{\text{ft}^3}{s}$$