SECTION 14.4 - SELECTED EVEN PROBLEMS

#6. (14.4)

$$z = \ln(x-2y) \quad ; \quad P(3,1,0)$$

$$\frac{\partial z}{\partial x} = \frac{1}{x-2y} \quad ; \quad \frac{\partial z}{\partial x}(3,1) = 1$$

$$\frac{\partial z}{\partial y} = \frac{-2}{x-2y} \quad ; \quad \frac{\partial z}{\partial y}(3,1) = -2$$

Tangent plane:

$$z - 0 = (x-3) - 2(y-1)$$

$$z = x - 2y - 1$$

$$x - 2y - z = 1$$

#14.

$$f(x,y) = \sqrt{x+e^{4y}} \quad ; \quad P(3,0)$$

$$f_x(x,y) = \frac{1}{2\sqrt{x+e^{4y}}}$$

$$f_y(x,y) = \frac{4e^{4y}}{2\sqrt{x+e^{4y}}} = \frac{2e^{4y}}{\sqrt{x+e^{4y}}}$$

$x + e^{4y} \neq 0$ when $x = 3, y = 0$

$\Rightarrow f_x, f_y$ are defined and continuous at $(3,0)$

(compositions and arithmetic operations with continuous functions.)

By the <u>Fundamental lemma on Differentiation</u>,

f is differentiable at $(3,0)$

$$f(3,0) = \sqrt{3+1} = 2$$

$$f_x(3,0) = \frac{1}{4} \quad ; \quad f_y(3,0) = 1$$

Linearization:

$$L(x,y) = 2 + \frac{1}{4}(x-3) + (y-0)$$

$$= \frac{1}{4}x + y + \frac{5}{4}.$$

#16.
(14.4)

$$f(x,y) = y + \sin\frac{x}{y} \quad ; \quad P(0,3)$$

$$f_x(x,y) = \frac{1}{y}\cos\frac{x}{y}$$

$$f_y(x,y) = 1 - \frac{x}{y^2}\cos\frac{x}{y}$$

$y \neq 0$ when $(x,y) = (0,3) \Rightarrow f_x, f_y$ are defined and continuous at $(0,3)$.

By the Fundamental Lemma on Differentiation, f is differentiable at $(0,3)$.

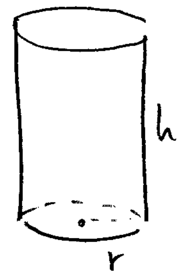
$$f(0,3) = 3$$

$$f_x(0,3) = \frac{1}{3} \quad ; \quad f_y(0,3) = 1$$

Linearization:

$$L(x,y) = 3 + \frac{1}{3}x + (y-3) = \frac{1}{3}x + y.$$

34.
(14.4)



$$V = \pi r^2 h$$

$$dV = 2\pi r h \, dr + \pi r^2 dh$$

$$= 2\pi \cdot 2 \cdot 10 \cdot \underbrace{0.05}_{\text{sides}} + \pi \cdot 2^2 \cdot \underbrace{0.2}_{\text{top and bottom}}$$

$$= 2\pi + 2\pi \cdot 0.4$$

$$= 2.8 \cdot \pi \approx 8.7965 \text{ cm}^3.$$

40.
(14.4)

$$f(x_1, x_2, x_3, x_4) = x_1 x_2 x_3 x_4$$

$$\Delta f = f(x_1 + \Delta x_1, x_2 + \Delta x_2, x_3 + \Delta x_3, x_4 + \Delta x_4) - f(x_1, x_2, x_3, x_4)$$

$$= f_{x_1} \Delta x_1 + f_{x_2} \Delta x_2 + f_{x_3} \Delta x_3 + f_{x_4} \Delta x_4 + \text{higher order terms}$$

$$|\Delta f| \lesssim f_{x_1} |\Delta x_1| + f_{x_2} |\Delta x_2| + f_{x_3} |\Delta x_3| + f_{x_4} |\Delta x_4|$$

higher order terms ignored

$$= x_2 x_3 x_4 |\Delta x_1| + x_1 x_3 x_4 |\Delta x_2| + x_1 x_2 x_4 |\Delta x_1| + x_1 x_2 x_3 |\Delta x_4|$$

$$\le 4 \cdot 50 \cdot 50 \cdot 50 \cdot 0.05$$

$$= 4 \cdot 25 \cdot 25 \cdot 10 = 25{,}000$$

This is a fairly accurate worst-case estimate.

Example: $50^4 - 49.95^4 = 24{,}963$

#42.
(14.4)

$\vec{r}_1(t) = (2+3t,\ 1-t^2,\ 3-4t+t^2)$

$\vec{r}_2(u) = (1+u^2,\ 2u^3-1,\ 2u+1)$

$P(2,1,3)$ corresponds to $t=0$, $u=1$

$\vec{r}_1{}'(t) = (3, -2t, -4+2t)$

– tangent vector to first curve

$\vec{r}_1{}'(0) = (3, 0, -4)$

$\vec{r}_2{}'(t) = (2u,\ 6u^2,\ 2)$

– tangent vector to 2nd curve

$\vec{r}_2{}'(1) = (2, 6, 2)$

Tangent plane contains both these vectors.

Normal vector:

$$\begin{vmatrix} i & j & k \\ 3 & 0 & -4 \\ 1 & 3 & 1 \end{vmatrix} = (12, -7, 9)$$

Tangent plane:

$$12(x-2) - 7(y-1) + 9(z-3) = 0$$

#46.

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

(a) $$f_x(0,0) = \lim_{h\to 0} \frac{\underbrace{f(0+h,0) - f(0,0)}_{=0}}{h} = 0$$

$$f_y(0,0) = \lim_{h\to 0} \frac{\underbrace{f(0,0+h) - f(0,0)}_{=0}}{h} = 0$$

since $f(x,y) = 0$ on the x-axis and the y-axis.

However $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ D.N.E.

(Since $x = y \Rightarrow f(x,y) = \frac{1}{2}$
$x = 0 \Rightarrow f(x,y) = 0$)
– different values along different directions of approach.

$\Rightarrow$ $f(x,y)$ is not continuous at $(0,0)$

$\Rightarrow$ $f(x,y)$ is not differentiable at $(0,0)$.

(b) $$f_x(x,y) = \frac{y}{x^2+y^2} - \frac{xy \cdot 2x}{(x^2+y^2)^2} = \frac{y(y^2-x^2)}{(x^2+y^2)^2}$$

$$f_y(x,y) = \frac{x}{x^2+y^2} - \frac{xy \cdot 2y}{(x^2+y^2)^2} = \frac{x(x^2-y^2)}{(x^2+y^2)^2}$$

For both functions $\displaystyle\lim_{(x,y)\to(0,0)}$ does not exist:

take $x = 0$ for f_x, $y = 0$ for f_y $\Rightarrow$

$$f_x(0,y) = \frac{1}{y} \xrightarrow[y \to 0\pm]{} \pm\infty$$

$$f_y(x,0) = \frac{1}{x} \xrightarrow[x \to 0\pm]{} \pm\infty$$