#12. (14.3)

$$f(x,y) = \sqrt{4-x^2-4y^2}$$

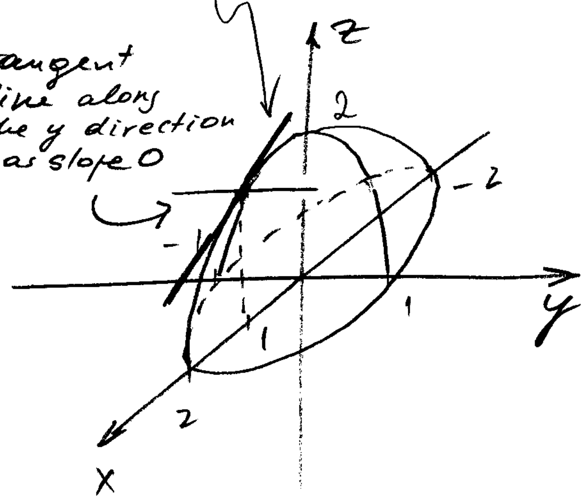
$$f_x(x,y) = \frac{-2x}{2\sqrt{4-x^2-4y^2}} = -\frac{x}{\sqrt{4-x^2-4y^2}}$$

$$f_y(x,y) = \frac{-8y}{2\sqrt{4-x^2-4y^2}} = -\frac{4y}{\sqrt{4-x^2-4y^2}}$$

tangent line along the x-direction has slope $-\frac{1}{\sqrt{3}}$

$$f_x(1,0) = -\frac{1}{\sqrt{3}} \; ; \quad f_y(1,0) = 0$$

tangent line along the y direction has slope 0



$$z = \sqrt{4-x^2-4y^2}$$

$$\iff z^2 = 4 - x^2 - 4y^2$$

$$\iff x^2 + 4y^2 + z^2 = 4$$

$$\iff \left(\frac{x}{2}\right)^2 + y^2 + \left(\frac{z}{2}\right)^2 = 1$$

– ellipsoid extending from -2 to 2 in the x-direction, -1 to 1 in the y-direction

$0 \leq z \leq 2$ because of the square root.

#18.

$$\frac{\partial f}{\partial x} = \frac{\ln t}{2\sqrt{x}} \; ; \quad \frac{\partial f}{\partial t} = \frac{\sqrt{x}}{t}$$

#28. (14.3)

$$x^y = e^{y\ln x}$$

$$\frac{\partial}{\partial x}(x^y) = \frac{\partial}{\partial x}\left(e^{y\ln x}\right) = e^{y\ln x} \cdot \frac{y}{x}$$

$$= x^y \cdot \frac{y}{x} = y x^{y-1}.$$

$$\frac{\partial}{\partial y}(x^y) = \frac{\partial}{\partial y}\left(e^{y\ln x}\right) = e^{y\ln x} \cdot \ln x$$

$$= \ln x \cdot x^y$$

#36. (14.3)

$$u = x^{y/z} = e^{\frac{y\ln x}{z}}$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} e^{\frac{y\ln x}{z}} = \frac{y}{zx} \cdot e^{\frac{y\ln x}{z}} = \frac{y \cdot x^{y/z}}{xz}$$

$$= \frac{y}{z} \cdot x^{\frac{y}{z}-1}$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y} e^{\frac{y\ln x}{z}} = \frac{\ln x}{z} \cdot e^{\frac{y\ln x}{z}} = \frac{\ln x}{z} \cdot x^{\frac{y}{z}}$$

$$\frac{\partial u}{\partial z} = \frac{\partial}{\partial z} e^{\frac{y\ln x}{z}} = -\frac{y\ln x}{z^2} e^{\frac{y\ln x}{z}} = -\frac{y\ln x}{z^2} \cdot x^{\frac{y}{z}}.$$

#42 (14.3)

$$f(x,y) = \arctan\frac{y}{x}$$

$$f_x(x,y) = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \left(-\frac{y}{x^2}\right) = -\frac{y}{x^2+y^2}$$

$$f_x(2,3) = -\frac{3}{2^2+3^2} = -\frac{3}{13}.$$

#76 (14.3)

(a) NO: $u_{xx} + u_{yy} = 4$.

(b) YES: $u_{xx} = 2$; $u_{yy} = -2$

(c) NO: $u_{xx} = 6x$; $u_{yy} = 6x$

(d) YES: $u = \frac{1}{2} \ln (x^2+y^2)$

$$u_x = \frac{x}{x^2+y^2} \; ; \; u_{xx} = \frac{1}{x^2+y^2} - \frac{2x^2}{(x^2+y^2)^2}$$

$$= \frac{y^2-x^2}{(x^2+y^2)^2}$$

Similarly,

$$u_{yy} = \frac{x^2-y^2}{(x^2+y^2)^2}.$$

(e) YES: $u_{xx} = -\sin x \cosh y - \cos x \sinh y$
$u_{yy} = \sin x \cosh y + \cos x \sinh y$

(f) YES: $u_{xx} = e^{-x} \cos y + e^{-y} \cos x$
$u_{yy} = -e^{-x} \cos y - e^{-y} \cos x$

#88 (14.3)

$$PV = mRT$$

$$P = \frac{mRT}{V} \Rightarrow \frac{\partial P}{\partial V} = -\frac{mRT}{V^2}$$

$$V = \frac{mRT}{P} \Rightarrow \frac{\partial V}{\partial T} = \frac{mR}{P}$$

$$T = \frac{PV}{mR} \Rightarrow \frac{\partial T}{\partial P} = \frac{V}{mR}$$

$$\frac{\partial P}{\partial V} \cdot \frac{\partial V}{\partial T} \cdot \frac{\partial T}{\partial P} = -\frac{mRT}{V^{\cancel{2}}} \frac{\cancel{mR}}{P} \frac{\cancel{V}}{\cancel{mR}} = -\frac{mRT}{PV} = -1.$$