MATH 250

SECTION 14.2 — SELECTED EVEN PROBLEMS

#6.

$$\lim_{(x,y)\to(1,-1)} e^{-xy}\cos(x+y) = e^{1}\cos(0) = e$$

(plug values in, using continuity.)

#8.

$$\lim_{(x,y)\to(1,0)} \ln\left(\frac{1+y^2}{x^2+xy}\right) = \ln 1 = 0.$$

#12.

$$\lim_{(x,y)\to(1,0)} \frac{xy-y}{(x-1)^2+y^2} = \lim_{(x,y)\to(1,0)} \frac{(x-1)y}{(x-1)^2+y^2}$$

$$\underset{\substack{\text{change}\\ x\leftarrow x-1}}{=} \lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

But this limit does not exist (See Example 2)

Answer: D.N.E.

#14.

$$\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)(x^2-y^2)}{(x^2+y^2)}$$

simplify using algebra.

$$= \lim_{(x,y)\to(0,0)} x^2-y^2 = 0$$

plug values in using continuity.

#16.

(14.2)

$$\frac{x^2 \sin^2 y}{x^2+2y^2} = \frac{x \sin^2 y}{\sqrt{2}y} \underbrace{\left(\frac{x \cdot \sqrt{2}y}{x^2+2y^2}\right)}_{-\frac{1}{2} \leq \cdot \leq \frac{1}{2}}$$

$$|\sin y| \leq |y| \Rightarrow \left|\frac{\sin y}{y}\right| \leq 1$$

$$-\frac{1}{2\sqrt{2}} \cdot 1 \cdot |x \sin y| \leq \frac{x^2 \sin^2 y}{x^2+2y^2} \leq \frac{1}{2\sqrt{2}} \cdot 1 \cdot |x \sin y|$$

Since $|x \sin y|$ is continuous at $(0,0)$

and $\lim_{(x,y)\to(0,0)} |x \sin y| = 0$, we have

by the Squeezing Principle,

$$\lim_{(x,y)\to(0,0)} \frac{x^2 \sin^2 y}{x^2+2y^2} = 0.$$

#18.

$$\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8} = \lim_{\substack{(x,y)\to(0,0)\\ y\geq 0}} \underbrace{\frac{xy}{x^2+y^2}}$$

This limit does not exist, as in Example 2.

OR: Try the function

$$f(x,y) = \frac{xy^4}{x^2+y^8}$$

on the curves $\left.\begin{array}{l}\{x=0\}\\ \{y=0\}\end{array}\right\}$ limit $=0$

$\{x=y^4\}$ } limit $=\frac{1}{2}$

$\Rightarrow$ the two-dimensional limit does not exist.

#20. (14.2)

$$xy + yz = 0 \quad \text{if} \quad \begin{array}{ll} x = y = 0 & (z\text{-axis}) \\ y = z = 0 & (x\text{-axis}) \\ x = z = 0 & (y\text{-axis}). \end{array}$$

However, if $x = y = z$,

then $$f(x,y,z) = \frac{xy+yz}{x^2+y^2+z^2} = \frac{2x^2}{3x^2} = \frac{2}{3} \quad (x \neq 0)$$

$$\Rightarrow \lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz}{x^2+y^2+z^2} \quad \text{Does not Exist.}$$

#38.

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+xy+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

is not continuous at $(0,0)$, since

$$\frac{xy}{x^2+xy+y^2} = 0 \quad \text{if either } x=0 \text{ or } y=0$$

but $$\frac{xy}{x^2+xy+y^2} = \frac{1}{3} \quad \text{if} \quad x = y.$$

$$\Rightarrow \lim_{(x,y)\to(0,0)} f(x,y) \quad \text{does not exist.}$$