#14.

$P(-2,4,0) \qquad Q(1,1,1)$

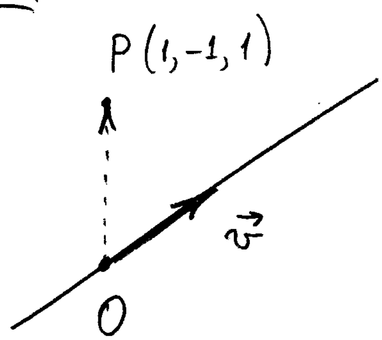
$R(2,3,4) \qquad S(3,-1,-8)$

$$\vec{PQ} = (3,-3,1) \ ; \quad \vec{RS} = (1,-4,-12)$$

$$\vec{PQ}\cdot\vec{RS} = 3+12-12 = 3 \neq 0$$

the lines are not perpendicular.

#36.



L : $x = 2y = 3z$

Pick a point on the line, for instance, the origin $O(0,0,0)$.

The directing vector of the line is $\vec{v} = \left(1, \frac{1}{2}, \frac{1}{3}\right)$.

Take the cross product (easier to work with vector $6\vec{v} = (6,3,2)$)

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 6 & 3 & 2 \end{vmatrix} = \vec{i}\begin{vmatrix} -1 & 1 \\ 3 & 2 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 1 \\ 6 & 2 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & -1 \\ 6 & 3 \end{vmatrix}$$

$$= -5\vec{i} + 4\vec{j} + 9\vec{k}$$

$\Rightarrow$ the plane is $\quad -5x + 4y + 9z = 0$

(it passes through the origin, as well as through $P(1,-1,1)$).

#48 (12.5)

Line: $\vec{r} = (x, y, z) = (1, 0, 1) + t(3, -2, 1)$

or $\begin{cases} x = 1+3t \\ y = -2t \\ z = 1+t \end{cases}$

Substitute into $x+y+z = 6$ to find the point of intersection:

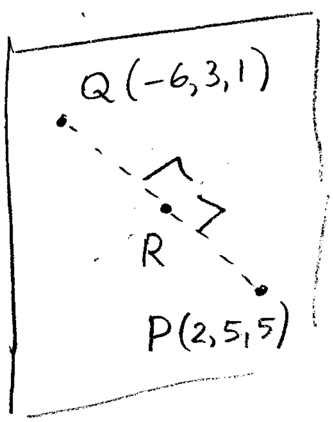
$1+3t - 2t + 1+t = 6$

$2t = 4$

$t = 2 \implies \begin{cases} x = 7 \\ y = -4 \\ z = 3 \end{cases}$

point of intersection $P(7, -4, 3)$.

#62 (12.5)



The plane has to pass through the mid-point and be perpendicular to $\overrightarrow{PQ}$.

$R\left(\frac{-6+2}{2}, \frac{3+5}{2}, \frac{1+5}{2}\right)$

$\overrightarrow{PQ} = (-8, -2, -4)$

$= -2(4, 1, 2)$

$\to R(-2, 4, 3)$

$\implies$ the plane is $4(x+2) + (y-4) + 2(z-3) = 0$

#68. (12.5)

L_1 and L_3 are parallel but not identical

L_2 and L_4 are identical.

#72. (12.5)

$$d = \frac{|ax+by+cz+d|}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{|-6-6-20-8|}{\sqrt{1^2+2^2+4^2}} = \frac{40}{\sqrt{21}}.$$

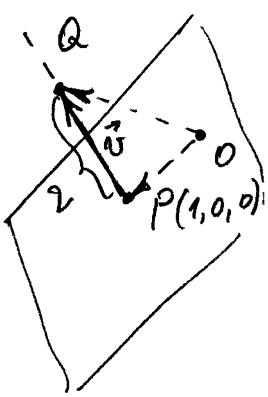
#76. (12.5)

$\vec{n} = (1, 2, -2)$ has length 3.

$\vec{v} = \frac{2}{3}\vec{n} = \left(\frac{2}{3}, \frac{4}{3}, -\frac{4}{3}\right)$ has same direction and length 2.

Pick a point on the plane; for instance

$P(1, 0, 0)$



$\vec{OP} + \vec{v} = \left(\frac{5}{3}, \frac{4}{3}, -\frac{4}{3}\right)$

points to a point Q, which is distance 2 away from the plane.

$$\Rightarrow \left(x - \frac{5}{3}\right) + 2\left(y - \frac{4}{3}\right) - 2\left(z + \frac{4}{3}\right) = 0$$

or $3x + 6y - 6z = 21$.