SECTION 12.4 — SELECTED EVEN PROBLEMS

#10.

$$\vec{k} \times (\vec{i} - 2\vec{j}) = \vec{k} \times \vec{i} - 2\vec{k} \times \vec{j}$$
$$= \vec{j} + 2\vec{i}$$

#12.

$$(\vec{i} + \vec{j}) \times (\vec{i} - \vec{j}) = \underbrace{\vec{i} \times \vec{i}}_{= \vec{0}} + \underbrace{\vec{j} \times \vec{i}}_{= -\vec{k}} - \underbrace{\vec{i} \times \vec{j}}_{= \vec{k}} - \underbrace{\vec{j} \times \vec{j}}_{= \vec{0}}$$

$$= -2\vec{k}$$

One way:

#20:

$$(\vec{j} - \vec{k}) \times (\vec{i} + \vec{j}) = \vec{i} - \vec{j} - \vec{k}$$

unit vectors: $\frac{1}{\sqrt{3}}\vec{i} - \frac{1}{\sqrt{3}}\vec{j} - \frac{1}{\sqrt{3}}\vec{k}$

and $-\frac{1}{\sqrt{3}}\vec{i} + \frac{1}{\sqrt{3}}\vec{j} + \frac{1}{\sqrt{3}}\vec{k}$.

Another way:

$$\vec{i} + \vec{j} = (1, 1, 0)$$
$$\vec{j} - \vec{k} = (0, 1, -1)$$

Guess a vector which gives dot product 0 with both vectors:

$$(1, -1, -1)$$

unit vectors: $\pm\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$.

#48.
(12.4)

$$\vec{a} + \vec{b} + \vec{c} = 0$$

$$\Rightarrow \quad \vec{c} = -\vec{a} - \vec{b}$$

$$\Rightarrow \quad \vec{b} \times \vec{c} = \vec{b} \times (-\vec{a} - \vec{b}) = -\vec{b} \times \vec{a} - \underbrace{\vec{b} \times \vec{b}}_{=0}$$

$$= \vec{a} \times \vec{b}.$$

Also $\quad \vec{a} = -\vec{b} - \vec{c}$

$$\Rightarrow \quad \vec{c} \times \vec{a} = \vec{c} \times (-\vec{b} - \vec{c}) = -\vec{c} \times \vec{b} - \underbrace{\vec{c} \times \vec{c}}_{=0}$$

$$= \vec{b} \times \vec{c}.$$