MATH 250

SECTIONS 12.2-3 SELECTED EVEN PROBLEMS.

#4: (12.2)

(a) $\overrightarrow{AC}$ (b) $\overrightarrow{CB}$

(c) $\overrightarrow{DA}$ (d) $\overrightarrow{DB}$

#26 (12.2)

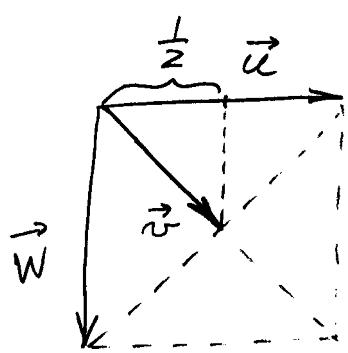
$\vec{a} = (-2, 4, 2)$

$|\vec{a}| = \sqrt{2^2+4^2+2^2} = \sqrt{24} = 2\sqrt{6}.$

$\vec{v} = 6\dfrac{\vec{a}}{|\vec{a}|} = \dfrac{6}{2\sqrt{6}}(-2, 4, 2) = \dfrac{\sqrt{6}}{2}(-2, 4, 2)$

$= \sqrt{6}(-1, 2, 1) = (-\sqrt{6}, 2\sqrt{6}, \sqrt{6}).$

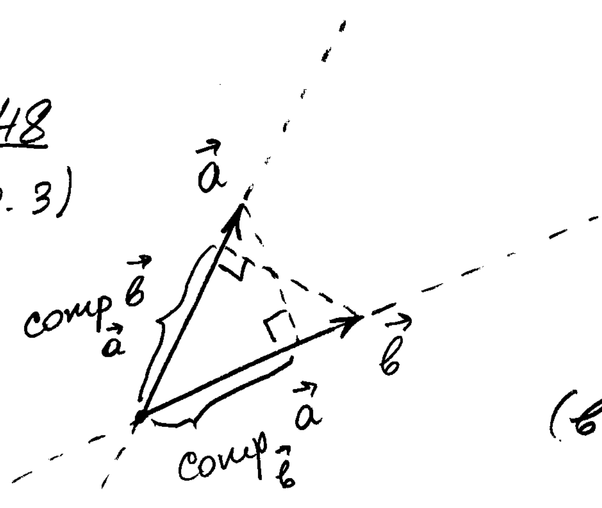
#12 (12.3)



$\vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{u} = \text{comp}_{\vec{u}}\vec{v} = \frac{1}{2}.$

$\vec{u}\cdot\vec{w} = 0$, since $\vec{u}$ and $\vec{w}$ are perpendicular.

#48 (12.3)



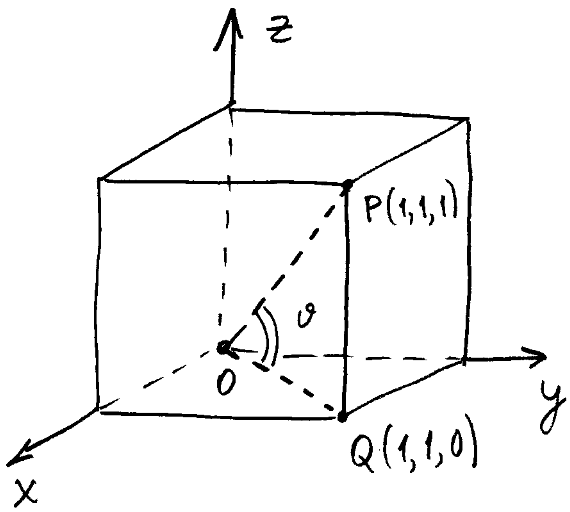
(a) if and only if $|\vec{a}| = |\vec{b}|$ (see figure.)

(b) $\text{proj}_{\vec{b}}\vec{a} = \text{proj}_{\vec{a}}\vec{b}$

$\Rightarrow$ $\vec{a}$ and $\vec{b}$ are parallel

by part (a), $|\vec{a}| = |\vec{b}| \Rightarrow \vec{a}$ and $\vec{b}$ are <u>equal</u>.

 # 56
(12.3)



$$\cos\theta = \frac{\vec{OP}\cdot\vec{OQ}}{|\vec{OP}|\,|\vec{OQ}|}$$

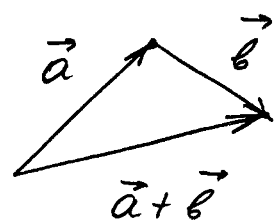
$$= \frac{1\cdot 1 + 1\cdot 1 + 1\cdot 0}{\sqrt{1^2+1^2+1^2}\,\sqrt{1^2+1^2+0^2}}$$

$$= \frac{2}{\sqrt{3}\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\theta = \cos^{-1}\frac{\sqrt{2}}{\sqrt{3}} \approx \underset{\text{(radians)}}{0.6155} \approx 35^{\circ}$$

 # 62
(12.3)

(a)



The length of any side in a triangle is always less than the sum of lengths of the remaining two sides.

(b)

$$|\vec{a}+\vec{b}|^2 = (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = |\vec{a}|^2 + \vec{b}\cdot\vec{a} + \vec{a}\cdot\vec{b} + |\vec{b}|^2$$

$$= |\vec{a}|^2 + 2\,\vec{a}\cdot\vec{b} + |\vec{b}|^2$$

Cauchy-Schwarz

$$\le |\vec{a}|^2 + 2\,|\vec{a}|\,|\vec{b}| + |\vec{b}|^2$$

$$= \left(|\vec{a}| + |\vec{b}|\right)^2$$

$$\Rightarrow |\vec{a}+\vec{b}| \le |\vec{a}| + |\vec{b}|.$$

64
(12.3)

$$(\vec{u}+\vec{v})\cdot(\vec{u}-\vec{v})=0$$

$$\vec{u}\cdot\vec{u}+\vec{v}\cdot\vec{u}-\vec{u}\cdot\vec{v}-\vec{v}\cdot\vec{v}=0$$

Cancel

$$|\vec{u}|^2-|\vec{v}|^2=0$$

$$|\vec{u}|^2=|\vec{v}|^2$$

$$|\vec{u}|=|\vec{v}|.$$