The essential guide for solving problems on local maxima and minima

Below is a summary of the <u>*Second Derivative Test*</u> (Theorem 3 in 14.7), used to solve problems in that section.

Given a function $z = f(x, y)$:

1. Compute the <u>first partials</u> $f_x(x, y)$ and $f_y(x, y)$.
2. Find <u>critical points</u> by solving the equations $f_x(x, y) = 0$, $f_y(x, y) = 0$.
3. Compute the <u>second partials</u> $f_{xx}(x, y)$, $f_{xy}(x, y)$, and $f_{yy}(x, y)$.
4. For each critical point (x_0, y_0), <u>compute the value $\Delta = AC - B^2$</u>, where $A = f_{xx}(x_0, y_0)$, $B = f_{xy}(x_0, y_0)$, $C = f_{yy}(x_0, y_0)$.

   (It is convenient to organize the values of the second derivatives into a 2×2 matrix:

   $$D^2 f = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix}, \text{ then } D^2 f(x_0, y_0) = \begin{pmatrix} A & B \\ B & C \end{pmatrix},$$

   and Δ is the determinant of $D^2 f(x_0, y_0)$.)
5. (a) If $\Delta > 0$ and $A > 0$ then the value $f(x_0, y_0)$ is a <u>local minimum</u>

   (b) If $\Delta > 0$ and $A < 0$ then the value $f(x_0, y_0)$ is a <u>local maximum</u>

   (c) If $\Delta < 0$ then (x_0, y_0) is a <u>saddle point</u> (neither a maximum or a minimum)

   (d) If $\Delta = 0$, the test is inconclusive.