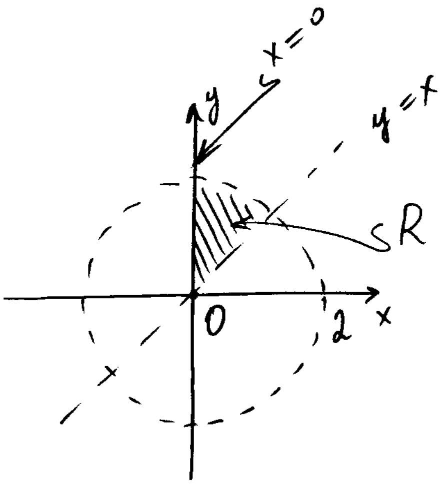
#8. $$\iint_R (2x-y)\,dx\,dy = \int_0^2 \int_{\pi/4}^{\pi/2} (2r\cos\theta - r\sin\theta)\,d\theta\, r\,dr$$



$x = r\cos\theta$
$y = r\sin\theta$
$dx\,dy = r\,dr\,d\theta$

$$= \int_0^2 r^2 \int_{\pi/4}^{\pi/2} (2\cos\theta - \sin\theta)\,d\theta\,dr$$

$$= \int_0^2 r^2\,dr \int_{\pi/4}^{\pi/2} (2\cos\theta - \sin\theta)\,d\theta$$

$$= \left[\frac{r^3}{3}\right]_0^2 \left[2\sin\theta + \cos\theta\right]_{\pi/4}^{\pi/2}$$

$$= \frac{8}{3}\left(2 - \frac{2}{\sqrt{2}} + 0 - \frac{1}{\sqrt{2}}\right)$$

$$= \frac{8}{3}\left(2 - \frac{3}{\sqrt{2}}\right).$$

#10. $$\iint_R \frac{y^2}{x^2+y^2}\,dx\,dy = \int_a^b r\,dr \int_0^{2\pi} \frac{r^2\sin^2\theta}{r^2\cos^2\theta + r^2\sin^2\theta}\,d\theta$$

$$R = \{(x,y):\ a \le \sqrt{x^2+y^2} \le b\}$$
$$= \{(x,y):\ a \le r \le b\}$$
$$(r = \sqrt{x^2+y^2})$$

$$= \frac{b^2-a^2}{2} \underbrace{\int_0^{2\pi} \sin^2\theta\,d\theta}_{=\pi} = \pi\,\frac{b^2-a^2}{2}$$

(half of the length of interval for 2π.)

2

12.

$$\iint_D \cos\sqrt{x^2+y^2}\,dx\,dy = \int_0^{2\pi} d\theta \int_0^2 dr \cdot (\cos r) \cdot r$$

$$D = \{(x,y):\ x^2+y^2 \le 4\}$$

$$= 2\pi \int_0^2 r\,(\sin r)'\,dr = 2\pi\left(\left[r\sin r\right]_0^2 - \int_0^2 \sin r\,dr\right)$$

$$= 2\pi\left(2\sin 2 + \left[\cos r\right]_0^2\right) = 2\pi\left(2\sin 2 + \cos 2 - 1\right).$$

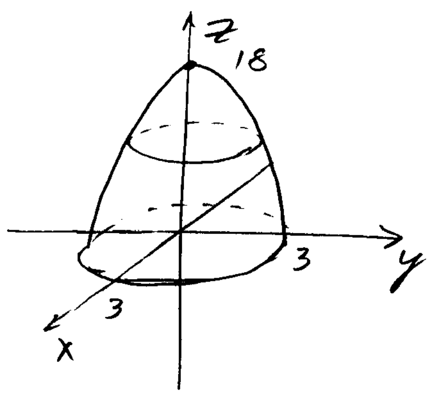
$$\approx 2.5287.$$

20.

$$z = 18 - 2x^2 - 2y^2$$

$$z = 0 \Rightarrow 2(x^2+y^2) = 18$$

$$\Rightarrow x^2+y^2 = 9$$



$$V = \iint_R 18 - 2x^2 - 2y^2\,dx\,dy$$

$$\left[R = \{(x,y):\ x^2+y^2 \le 9\}\right]$$

$$= \int_0^{2\pi} d\theta \int_0^3 (18 - 2r^2)\,r\,dr$$

$$= 2\pi\left(18\left[\frac{r^2}{2}\right]_0^3 - 2\left[\frac{r^4}{4}\right]_0^3\right)$$

$$= 2\pi\left(3^4 - \frac{3^4}{2}\right) = \pi \cdot 3^4 = 81\pi.$$