SECTION 15.3 – SELECTED EVEN PROBLEMS

#6 (15.3)

$$\int_0^1 \int_0^{e^v} \sqrt{1+e^v}\, dw\, dv = \int_0^1 \sqrt{1+e^v}\, e^v\, dv$$

$$= \int_0^e \sqrt{1+u}\, du = \frac{2}{3}\Big[(1+u)^{3/2}\Big]_{u=0}^{u=e} = \frac{2}{3}\left((1+e)^{3/2}-1\right)$$

#8. (15.3)

$$\iint_D \frac{y}{x^5+1}\, dx\, dy = \int_0^1 \int_0^{x^2} \frac{y}{x^5+1}\, dy\, dx =$$

$D = \{(x,y):\ 0 \le x \le 1,\ 0 \le y \le x^2\}$

$$= \int_0^1 \frac{1}{x^5+1}\left[\frac{y^2}{2}\right]_0^{x^2} dx = \frac{1}{2}\underbrace{\int_0^1 \frac{x^4\, dx}{x^5+1}}$$

$u = x^5$
$du = 5x^4 dx$
$x^4 dx = \frac{1}{5} du$

$$= \frac{1}{2\cdot 5}\int_0^1 \frac{du}{u+1}$$

$$= \frac{1}{10}\Big[\ln(u+1)\Big]_{u=0}^{u=1} = \frac{\ln 2}{10}.$$

#10. (15.3)

$$\iint_D x^3\, dx\, dy = \int_1^e \int_0^{\ln x} x^3\, dy\, dx = \int_1^e x^3 \int_0^{\ln x} dy\, dx$$

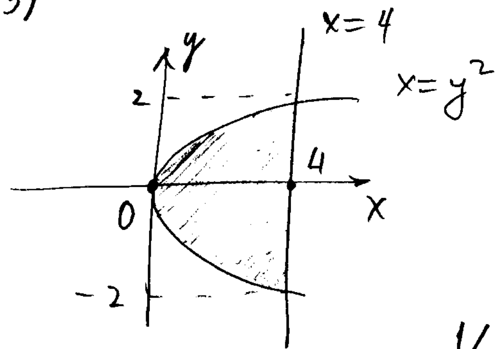
$D = \{(x,y):\ 1 \le x \le e,\ 0 \le y \le \ln x\}$

$$= \int_1^e x^3 \ln x\, dx = \int_1^e \left(\frac{1}{4}x^4\right)' \ln x\, dx = \left[\frac{1}{4}x^4 \ln x\right]_1^e - \int_1^e \frac{1}{4}x^4\cdot\frac{1}{x}\, dx$$

$$= \frac{1}{4}e^4 - \frac{1}{4}\left[\frac{x^4}{4}\right]_1^e = \frac{4e^4 - e^4 + 1}{16} = \frac{3e^4+1}{16}.$$

#24 (15.3)

$z = 1 + x^2y^2$ – top surface



$$R = \{(x,y) : -2 \le y \le 2,\ y^2 \le x \le 4\}$$

$$V = \int_{-2}^{2} \int_{y^2}^{4} (1 + x^2y^2)\,dx\,dy$$

$$= \int_{-2}^{2} (4 - y^2) + y^2 \left[\frac{x^3}{3}\right]_{y^2}^{4} dy$$

$$= \int_{-2}^{2} 4 - y^2 + \frac{1}{3} y^2 (64 - y^6)\,dy$$

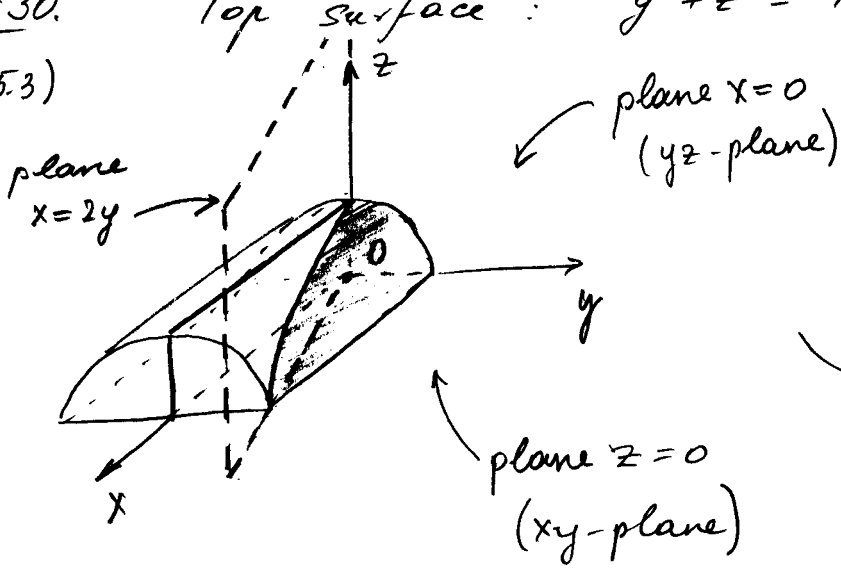
$$= 16 - \left[\frac{y^3}{3}\right]_{-2}^{2} + \frac{64}{3}\left[\frac{y^3}{3}\right]_{-2}^{2} - \frac{1}{3}\left[\frac{y^9}{9}\right]_{-2}^{2}$$

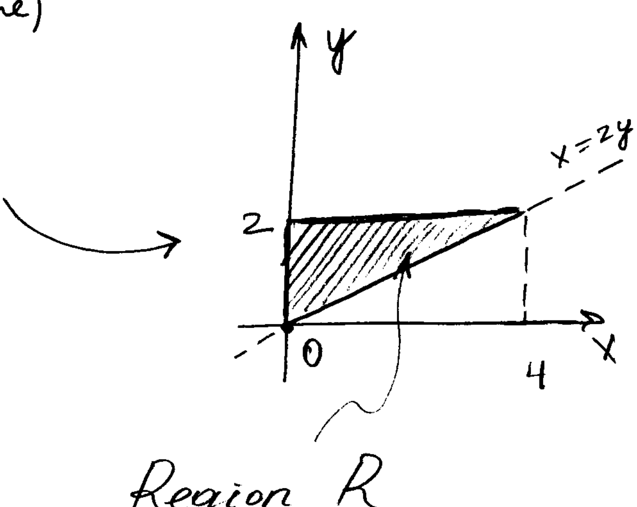
$$= 16 + \frac{61}{9} \cdot 16 - \frac{1}{27} \cdot 2 \cdot 512$$

$$= 16\left(1 + \frac{61}{9} - \frac{64}{27}\right) = \frac{2336}{27}.$$

#30. (15.3)

Top surface: $y^2 + z^2 = 4$, $z \ge 0 \Rightarrow z = \sqrt{4 - y^2}$.





$$V = \int_0^2 \int_0^{2y} \sqrt{4-y^2}\, dx\, dy = \int_0^2 \sqrt{4-y^2} \underbrace{\int_0^{2y} dx}_{=2y}\, dy$$

$$= \int_0^2 2y\sqrt{4-y^2}\, dy = \int_0^4 \sqrt{4-u}\, du = \left[-\frac{2}{3}(4-u)^{3/2}\right]_{u=0}^{u=4}$$

$y^2 = u$

$2y\,dy = du$

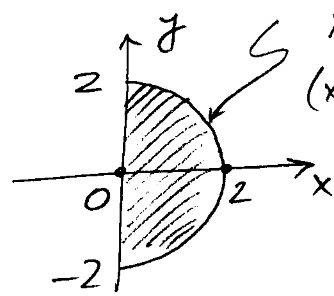
$$= \frac{2}{3}\cdot 4^{3/2} = \frac{16}{3}.$$

#46. (15.3) $\displaystyle\int_{-2}^{2} \int_0^{\sqrt{4-y^2}} f(x,y)\, dx\, dy$

$0 \le x \le \sqrt{4-y^2}$

$-2 \le y \le 2$ $\Rightarrow$

half-circle $x = \sqrt{4-y^2}$ $(x^2+y^2=4,\ x \ge 0)$



Then $0 \le x \le 2$

$-\sqrt{4-x^2} \le y \le \sqrt{4-x^2}$

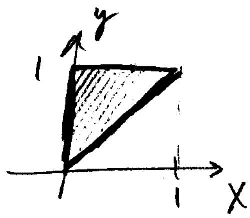
Change the order of integration:

$$\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x,y)\, dy\, dx.$$

#52. (15.3) $\displaystyle\int_0^1 \int_x^1 e^{x/y}\, dy\, dx = \int_0^1 \int_0^y e^{x/y}\, dx\, dy$

$0 \le x \le 1$

$x \le y \le 1$ $\Rightarrow$



$$= \int_0^1 \left[y e^{x/y}\right]_{x=0}^{x=y} dy$$

$$= \int_0^1 y(e-1)\, dy = (e-1)\left[\frac{y^2}{2}\right]_{y=0}^{y=1}$$

$$= \frac{e-1}{2}.$$