SECTION 15.2 - SELECTED EVEN PROBLEMS

#12. (15.2)

$$\int_0^1\int_0^1 xy\sqrt{x^2+y^2}\,dy\,dx = \int_0^1 x \underbrace{\int_0^1 \sqrt{x^2+y^2}\,y\,dy}_{\substack{u=y^2,\\ du=2y\,dy}}\,dx$$

$$= \int_0^1 x\,\frac{1}{2}\int_0^1 \sqrt{x^2+u}\,du\,dx = \frac{1}{2}\int_0^1 x\left[\frac{2}{3}(x^2+u)^{\frac{3}{2}}\right]_{u=0}^{u=1} dx$$

$$= \frac{1}{3}\int_0^1 x\left(\underbrace{(x^2+1)}_{\substack{v=x^2\\ dv=2x\,dx}}{}^{3/2} - x^3\right)dx = \frac{1}{6}\int_0^1 (v+1)^{3/2}dv - \frac{1}{3}\int_0^1 x^4 dx$$

$$= \frac{1}{6}\cdot\frac{2}{5}\cdot\left[(v+1)^{5/2}\right]_{v=0}^{v=1} - \frac{1}{3}\cdot\left[\frac{x^5}{5}\right]_0^1 = \frac{1}{15}\left(2^{5/2}-2\right)$$

#14. (15.2)

$$\int_0^1\int_0^1 \sqrt{s+t}\,ds\,dt = \int_0^1\left[\frac{2}{3}(s+t)^{3/2}\right]_{s=0}^{s=1} dt$$

$$= \frac{2}{3}\int_0^1 (t+1)^{3/2} - t^{3/2}\,dt = \frac{2}{3}\cdot\frac{2}{5}\left[(t+1)^{5/2}\right]_{t=0}^{t=1} - \frac{2}{3}\cdot\frac{2}{5}\cdot\left[t^{5/2}\right]_{t=0}^{t=1}$$

$$= \frac{4}{15}\left(2^{5/2} - 1^{5/2} - 1^{5/2}\right) = \frac{4}{15}\left(2^{5/2}-2\right) = \frac{8}{15}\left(2\sqrt{2}-1\right)$$

#38. (15.2)

$$\iint_R \left(1 + x^2 \sin y + y^2 \sin x\right) dx\,dy$$

$$R = [-\pi, \pi] \times [-\pi \times \pi].$$

$$= \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} 1 + \underbrace{x^2 \sin y}_{=0} + \underbrace{y^2 \sin x}_{=0} \, dx\, dy$$

since $\int_{-\pi}^{\pi} \sin y \, dy = 0$

Since $\int_{-\pi}^{\pi} \sin x \, dx = 0$

$$= \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} 1 \cdot dx\,dy = (2\pi)^2 = 4\pi^2.$$