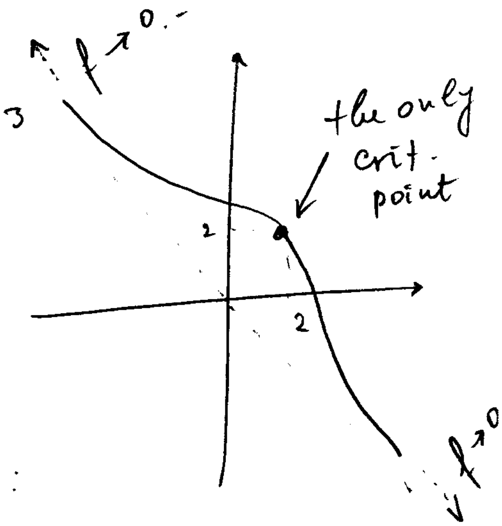
SECTION 14.8 - SELECTED EVEN PROBLEMS

#6. (14.8)

$f(x,y) = e^{xy}$, $x^3+y^3=16$

$g(x,y) = x^3+y^3$



$\vec{\nabla} f(x,y) = \left(ye^{xy},\ xe^{xy}\right)$

$\vec{\nabla} g(x,y) = \left(3x^2,\ 3y^2\right)$

Lagrange's multiple equations:

$$\begin{cases} ye^{xy} = 3\lambda x^2 \\ xe^{xy} = 3\lambda y^2 \\ x^3+y^3=16 \end{cases}$$

$\Rightarrow \frac{y}{x} = \frac{x^2}{y^2} \Rightarrow y^3 = x^3$

$\Rightarrow y = x$

$\Rightarrow 2x^3 = 16$

$x^3 = 8$

$x = 2,\ y = 2.$

Absolute max: $f(2,2) = e^4$.

$\Big(f(x,y) \geq 0,\quad f(x,y) \to 0$ as $x \to \infty$ or $x \to -\infty$ subj. to $x^3+y^3=16$ $\Rightarrow f(2,2)$ is a maximum $\Big)$

#16. (14.8)

$f(x,y,z) = 3x - y - 3z$

$x+y-z=0$, $x^2+2z^2=1$

$\vec{\nabla} f = (3, -1, -3)$

$\vec{\nabla} g = (1, 1, -1)$

$\vec{\nabla} h = (2x, 0, 4z)$

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Lagrange multiple equations:

$$\vec{\nabla} f = \lambda \vec{\nabla} g + \mu \vec{\nabla} h$$

$$\left.\begin{aligned} \lambda + 2x\mu &= 3 \\ \lambda &= -1 \\ -\lambda + 4z\mu &= -3 \end{aligned}\right\} \Rightarrow \begin{aligned} 2x\mu &= 4 \\ 4z\mu &= -4 \end{aligned}$$

$$\Rightarrow \frac{2x}{4z} = -1$$

$$x = -2z$$

Constraints:

$$\left.\begin{aligned} x + y - z &= 0 \\ x^2 + 2z^2 &= 1 \end{aligned}\right\} \Rightarrow 4z^2 + 2z^2 = 1$$

$$\Rightarrow 6z^2 = 1$$

$$\Rightarrow z = \frac{1}{\sqrt{6}}.$$

$$\Rightarrow x = -\frac{2}{\sqrt{6}}$$

$$y = z - x = \frac{3}{\sqrt{6}}$$

#18. (14.8)

$$f(x,y,z) = x^2 + y^2 + z^2$$

$$x - y = 1, \quad y^2 - z^2 = 1$$

$$\vec{\nabla} f = (2x, 2y, 2z)$$

$$\vec{\nabla} g = (1, -1, 0)$$

$$\vec{\nabla} h = (0, 2y, -2z)$$

Lagrange multiple equations

$$\vec{\nabla} f = \lambda \vec{\nabla} g + \mu \vec{\nabla} h$$

$$\left.\begin{aligned} \lambda &= 2x \\ -\lambda + 2y\mu &= 2y \\ -2z\mu &= 2z \end{aligned}\right\} \quad \begin{aligned} &\Rightarrow \lambda = 2x \\ \\ &\Rightarrow \mu = -1 \text{ or } z = 0 \end{aligned}$$

If $z = 0$ then $y^2 - z^2 = 1 \Rightarrow y = 1$

then $x - y = 1 \Rightarrow x = 2$

then $-2x + 2y\mu = 2y$

$\Rightarrow \mu = 3$

$\Rightarrow$ $\boxed{x = 2,\ y = 1,\ z = 0}$ is an extremum point.

If $\lambda = 2x$, $\mu = -1$ then

$-2x - 2y = 2y$

$\Rightarrow -2x = 4y \Rightarrow x = -2y$

or $y = -\frac{1}{2}x$

Solve the constraint equations:

$$\left.\begin{aligned} x + y - z &= 0 \\ x^2 + 2z^2 &= 1 \end{aligned}\right\} \Rightarrow \frac{1}{2}x - z = 0 \Rightarrow x - 2z = 0$$

$\Rightarrow 4z^2 + 2z^2 = 1 \Rightarrow 6z^2 = 1$

$z = \frac{1}{\sqrt{6}}$

Then $x = \frac{2}{\sqrt{6}}$, $y = -\frac{1}{\sqrt{6}}$ } extremum point.