#4 (14.7) $f(x,y) = 3x - x^3 - 2y^2 + y^4$

Predict: $P(-1,1)$ - local min

$Q(-1,-1)$ - local min

$R(-1,0)$ - saddle point

$S(1,0)$ - local max

$T(1,1)$ - saddle point

$U(1,-1)$ - saddle point.

Compute:

$$\vec{\nabla} f(x,y) = (3-3x^2, -4y+4y^3)$$

Critical pts: $\begin{cases} 3-3x^2=0 \\ 4y^3-4y=0 \end{cases} \Leftrightarrow \begin{cases} x^2=1 \\ y(y^2-1)=0 \end{cases} \Leftrightarrow \begin{cases} x=\pm 1 \\ y=0,\pm 1 \end{cases}$

Hessian matrix:

$$D^2 f(x,y) = \begin{pmatrix} -6x & 0 \\ 0 & -4+12y^2 \end{pmatrix}$$

$P(-1,1) \Rightarrow \Delta = 6\cdot 9 > 0$, $f_{xx} > 0 \Rightarrow$ local min

$Q(-1,-1) \Rightarrow \Delta = 6\cdot 9 > 0$, $f_{xx} > 0 \Rightarrow$ local min

$R(-1,0) \Rightarrow \Delta = 6\cdot(-4) < 0$, $\Rightarrow$ saddle pt.

$S(1,0) \Rightarrow \Delta = (-6)(-4) > 0$, $f_{xx} < 0 \Rightarrow$ local max

$T(1,1) \Rightarrow \Delta = (-6)(9) < 0 \Rightarrow$ saddle point

$U(1,-1) \Rightarrow \Delta = (-6)\cdot 9 < 0 \Rightarrow$ saddle point.

#6 (14.7)

$f(x,y) = xy - 2x - 2y - x^2 - y^2$

$\vec{\nabla} f(x,y) = (y - 2 - 2x,\ x - 2 - 2y)$

Critical points: $\begin{cases} y - 2x - 2 = 0 \\ x - 2y - 2 = 0 \end{cases} \iff \begin{matrix} x = -2 \\ y = -2 \end{matrix}$

$$D^2 f(x,y) = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}$$

$\Delta = 3 > 0$, $f_{xx} < 0 \implies P(-2,-2)$ is a local max.

#12. (14.7)

$f(x,y) = xy + \frac{1}{x} + \frac{1}{y}$

$\vec{\nabla} f(x,y) = \left(y - \frac{1}{x^2},\ x - \frac{1}{y^2} \right)$

Critical points: $\begin{cases} y - \frac{1}{x^2} = 0 \\ x - \frac{1}{y^2} = 0 \end{cases}$

$y = \frac{1}{x^2}$; $\frac{1}{y^2} = x^4$

$x - x^4 = 0$

$x(1 - x^3) = 0$

$x = 0$ or $x = 1$

Similarly, $y = 0$ or $y = 1$.

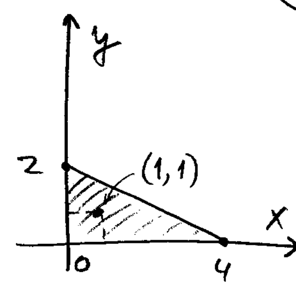
$x = 0$, or $y = 0$ are not crit. points, since f is not

$$D^2 f(x,y) = \begin{pmatrix} \frac{2}{x^3} & 1 \\ 1 & \frac{2}{y^3} \end{pmatrix}$$

$P(1,1) \implies \Delta = 4 - 1 > 0$, $f_{xx} > 0 \implies$ local min

#30.
(14.7)

$f(x,y) = x + y - xy$



$\nabla f(x,y) = (1-y,\ 1-x)$

critical point $\quad x=1,\ y=1$

$D^2 f(x,y) = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \Rightarrow \Delta = -1 < 0 \Rightarrow$ saddle point.

x-axis $\Rightarrow y = 0 \Rightarrow f(x,0) = x \Rightarrow$ min: $f(0,0) = 0$
max: $f(4,0) = 4$

y-axis $\Rightarrow x = 0 \Rightarrow f(0,y) = y \Rightarrow$ min: $f(0,0) = 0$
max: $f(0,2) = 2$

the line $\quad 2y + x = 4$

$\Rightarrow x = 4 - 2y \quad f(4-2y, y) = 4 - 2y + y - (4-2y)y$
$= 4 - 5y + 2y^2$

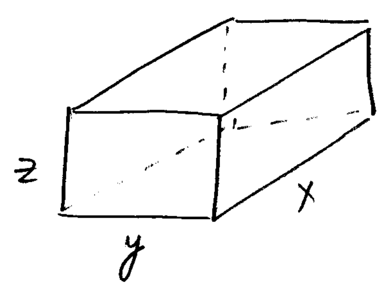
Solve $\quad z = 4 - 5y + 2y^2$ for max/min:

crit. pt: $\quad -5 + 4y = 0 \Rightarrow y = \frac{5}{4}$; $z = 4 - \frac{25}{4} + \frac{25}{8} = \frac{7}{8}$

min: $f(\frac{3}{2}, \frac{5}{4}) = \frac{7}{8}$
max: $f(4,0) = 4$

Absolute max: $f(4,0) = 4$
Absolute min: $f(0,0) = 0$.

#50
(14.7)



Cost function:

$f(x,y,z) = 5xy + 2yz + 2xz$

Volume: $\quad V = xyz$

$\Rightarrow z = \frac{V}{xy}$

$f(x,y) = 5xy + 2(x+y)\frac{V}{xy} = 5xy + \frac{2V}{y} + \frac{2V}{x}$

$$\vec{\nabla} f(x,y) = \left(5y - \frac{2V}{x^2}\ ,\ 5x - \frac{2V}{y^2}\right)$$

Critical points: $\quad 5y = \frac{2V}{x^2}\ ,\quad 5x = \frac{2V}{y^2}$

$$\Rightarrow\ 5x = 2V\left(\frac{5x^2}{2V}\right)^2 = \frac{25x^4}{2V}$$

$$x\left(1 - \frac{5x^3}{2V}\right) = 0$$

$$\underbrace{x=0}_{\text{volume}=0} \quad \text{or} \quad x = \left(\frac{2V}{5}\right)^{1/3}$$

$$\Rightarrow \qquad x = \left(\frac{2V}{5}\right)^{1/3},\ \ y = \left(\frac{2V}{5}\right)^{1/3},\ z = \left(\frac{25V}{4}\right)^{1/3}$$