

#4
(14.6)

$$f(x, y) = x^3 y^4 + x^4 y^3; \quad P(1, 1), \quad \vartheta = \frac{\pi}{6}$$

$$f_x(x, y) = 3x^2 y^4 + 4x^3 y^3; \quad f_x(1, 1) = 7$$

$$f_y(x, y) = 4x^3 y^3 + 3x^4 y^2; \quad f_y(1, 1) = 7$$

$$\begin{aligned} \vec{u} f(1, 1) &= f_x(1, 1) \cos \vartheta + f_y(1, 1) \sin \vartheta \\ &= 7(\cos \vartheta + \sin \vartheta) = 7\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) \\ &= \frac{7(\sqrt{3} + 1)}{2}. \end{aligned}$$

#16.
(14.6)

$$f(x, y, z) = \sqrt{xyz}; \quad P(3, 2, 6), \quad \vec{v} = (-1, -2, 2)$$

$$\vec{\nabla} f(x, y, z) = \left(\frac{\sqrt{yz}}{2\sqrt{x}}, \frac{\sqrt{xz}}{2\sqrt{y}}, \frac{\sqrt{xy}}{2\sqrt{z}} \right)$$

$$\vec{\nabla} f(3, 2, 6) = \left(\frac{\sqrt{12}}{2\sqrt{3}}, \frac{\sqrt{18}}{2\sqrt{2}}, \frac{\sqrt{6}}{2\sqrt{6}} \right) = \left(1, \frac{3}{2}, \frac{1}{2} \right).$$

$$\begin{aligned} \vec{v} \cdot \vec{\nabla} f(3, 2, 6) &= \left(1, \frac{3}{2}, \frac{1}{2} \right) \cdot (-1, -2, 2) \\ &= -1 - 3 + 1 = -3 \end{aligned}$$

#28.
(14.6)

$$f(x, y) = y e^{-xy}$$

$$\vec{\nabla} f(x, y) = (-y^2 e^{-xy}, e^{-xy} - xy e^{-xy})$$

$$\vec{\nabla} f(0, 2) = (-4, 1)$$

$$\begin{aligned} \vec{u} \cdot \vec{\nabla} f(0, 2) &= -4u_1 + u_2 = 1 \\ u_1^2 + u_2^2 &= 1 \end{aligned}$$

(2)

$$u_2 = 1 + 4u_1$$

$$u_1^2 + (1 + 4u_1)^2 = 1$$

$$u_1^2 + 1 + 8u_1 + 16u_1^2 = 1$$

$$8u_1 + 17u_1^2 = 0$$

$$u_1(8 + 17u_1) = 0$$

$$u_1 = 0 \quad \text{or} \quad u_1 = -\frac{8}{17} \quad \checkmark$$

$$\vec{u} = (0, 1) \quad \text{or} \quad \vec{u} = \left(-\frac{8}{17}, -\frac{15}{17}\right)$$