Thu, Nov 13, 2014 **MATH 250, Quiz 6** Prof. V. Panferov

Name (print): Solutions.

Each problem is worth 2 points. Show all your work.

1. The function $f(x,y) = xy - 2x - 2y - x^2 - y^2$ has a critical point at $(-2,-2)$. Determine its type (local maximum/minimum, saddle point...).

$$\vec{\nabla} f = (y - 2 - 2x,\ x - 2 - 2y)$$

$$D^2 f = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} A & B \\ B & C \end{pmatrix}$$

$$\Delta = AC - B^2 = 4 - 1 = 3 > 0$$

$$A = -2 < 0$$

$\Rightarrow$ $f(x,y)$ has a local maximum at the crit. pt. $(-2,2)$.

2. Find all critical points of the function

$$f(x,y) = y^2 - 2y\cos x.$$

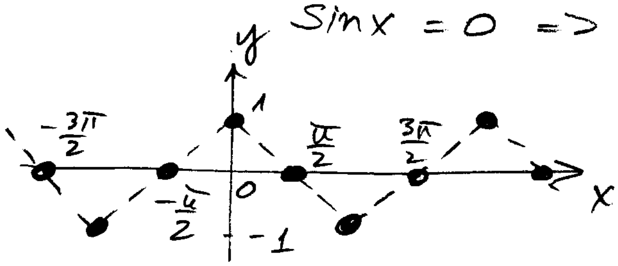
$$\vec{\nabla} f = (2y\sin x,\ 2y - 2\cos x) = (0,0)$$

$$\begin{cases} 2y\sin x = 0 \\ 2(y - \cos x) = 0 \end{cases} \Rightarrow \begin{cases} y = 0 \text{ or } \sin x = 0 \\ y = \cos x \end{cases}$$

$y = 0 \Rightarrow \cos x = 0 \Rightarrow x = \frac{\pi}{2} + \pi n$, $n = 0, \pm 1, \pm 2 \ldots$

$\sin x = 0 \Rightarrow x = \pi n \Rightarrow y = \cos(\pi n) = (-1)^n$

$n = 0, \pm 1, \pm 2 \ldots$



Answer:

$(\pi(n + \frac{1}{2}), 0)$

$(2\pi n, 1)$, $(2\pi(n + \frac{1}{2}), -1)$; $n = 0, \pm 1, \pm 2 \ldots$

Please turn over...

3. Set up the Lagrange Multiplier method equations for the problem. Show that the number of equations matches the number of unknowns. Do not solve the actual equations.

$$\begin{cases} f(x,y,z) = yz + xy \longrightarrow \max \\ g(x,y) = xy = 1, \quad h(x,y,z) = y^2 - z^2 = 1 \quad \text{(constraints)}. \end{cases}$$

$$\vec{\nabla} f = (y, z+x, y)$$

$$\vec{\nabla} g = (y, x, 0)$$

$$\vec{\nabla} h = (0, 2y, -2z)$$

Equations to solve for critical pts.:

$$\vec{\nabla} f = \lambda \vec{\nabla} g + \mu \vec{\nabla} h$$

$$\Rightarrow \left.\begin{cases} y = \lambda y \\ z + x = \lambda x + 2\mu y \\ y = -2\mu z \end{cases}\right.$$

+ two Constraints: $xy = 1$, $y^2 - z^2 = 1$

5 equations, 5 unknowns: x, y, z, λ, μ.