Tue, Sept. 16, 2014 **MATH 250, Quiz 2** Prof. V. Panferov

Name (print): *Solutions.*

Each problem is worth 2 points. Show all your work.

1. If $\vec{a} = (2, 1, -3)$ and $\vec{b} = (4, 2, 1)$, find $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -3 \\ 4 & 2 & 1 \end{vmatrix} = \vec{i}\begin{vmatrix} 1 & -3 \\ 2 & 1 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & -3 \\ 4 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix}$$

$$= 7\vec{i} - 14\vec{j} + 0\vec{k} = (7, -14, 0).$$

$$\vec{b} \times \vec{a} = -\vec{a} \times \vec{b} = (-7, 14, 0) = -7\vec{i} + 14\vec{j}.$$

2. Use properties of the cross product to prove that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$.

$$(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = (\vec{a} - \vec{b}) \times \vec{a} + (\vec{a} - \vec{b}) \times \vec{b}$$

distributive prop.

$$= (\underbrace{\vec{a} \times \vec{a}}_{=0} - \vec{b} \times \vec{a}) + (\vec{a} \times \vec{b} - \underbrace{\vec{b} \times \vec{b}}_{=0})$$

distributive prop.; factor (-1)

$$= -\vec{b} \times \vec{a} + \vec{a} \times \vec{b}$$

$$= \vec{a} \times \vec{b} + \vec{a} \times \vec{b} = 2\vec{a} \times \vec{b}.$$

anti-commutative prop.

Please turn over...

3. Find the point in $\mathbb{R}^3$ at which the line $x = y - 1 = 2z$ intersects the plane $4x - y + 3z = 8$.

$$\begin{cases} x = y - 1 = 2z \\ 4x - y + 3z = 8 \end{cases} \Rightarrow \begin{cases} x = 2z \\ y = 2z + 1 \\ 4x - y + 3z = 8z - 2z - 1 + 3z \\ \quad = 9z - 1 = 8 \end{cases}$$

$$\Rightarrow \quad \begin{aligned} 9z &= 9 \Rightarrow z = 1 \\ x &= 2 \\ y &= 3 \end{aligned}$$

$(x, y, z) = (2, 3, 1)$ — point of intersection.

4. Find an equation for the plane in $\mathbb{R}^3$ consisting of all points that are equidistant from the points $A(1, 0, -2)$ and $B(3, 4, 0)$.

One way:

$$(x-1)^2 + y^2 + (z+2)^2 = (x-3)^2 + (y-4)^2 + z^2$$

$$\cancel{x^2} - 2x + 1 + \cancel{y^2} + \cancel{z^2} + 4z + 4 = \cancel{x^2} - 6x + 9 + \cancel{y^2} - 8y + 16 + \cancel{z^2}$$

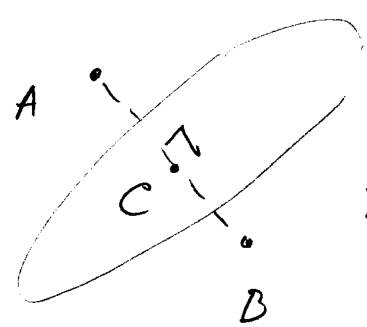
$$4x + 8y + 4z = 20$$

$$x + 2y + z = 5.$$

Another way:

Mid-point: $C(2, 2, -1)$

$\vec{AB} = (2, 4, 2)$



The plane must go through the mid-point and have $\vec{AB}$ as normal vector.

Plane: $2(x-2) + 4(y-2) + 2(z+1) = 0$

OR

$x + 2y + z = 5.$