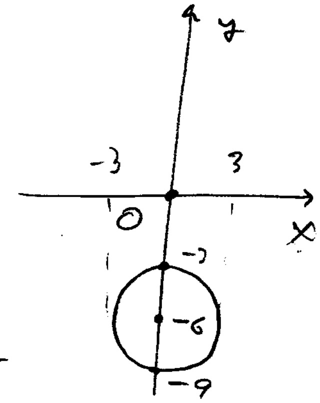
Homework for Sections 10.5, 12.6 – Solutions.

1 (a)

$$x^2+(y^2+12y+36)-36+27=0$$

$$x^2+(y+6)^2-9=0$$

$$\frac{x^2}{9}+\frac{(y+6)^2}{9}=1 \quad \text{– circle}$$

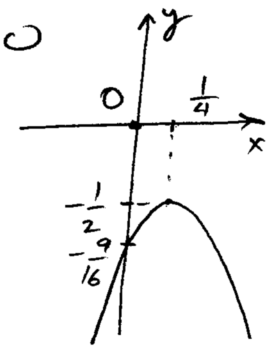


(b)

$$2\left(x^2-\frac{1}{2}x+\frac{1}{16}\right)-\frac{1}{8}+2y+\frac{9}{8}=0$$

$$2\left(x-\frac{1}{4}\right)^2+2y+1=0$$

$$\left(x-\frac{1}{4}\right)^2=-\left(y+\frac{1}{2}\right) \quad \text{– parabola}$$

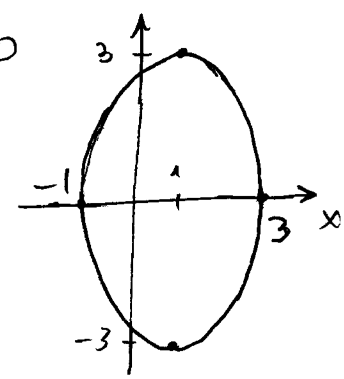


(c)

$$9(x^2-2x+1)-9+4y^2-27=0$$

$$9(x-1)^2+4y^2-36=0$$

$$\frac{(x-1)^2}{4}+\frac{y^2}{9}=1 \quad \text{– ellipse}$$



(d)

$$(x^2+6x+9)-9-2(y^2-2y+1)+2+5=0$$

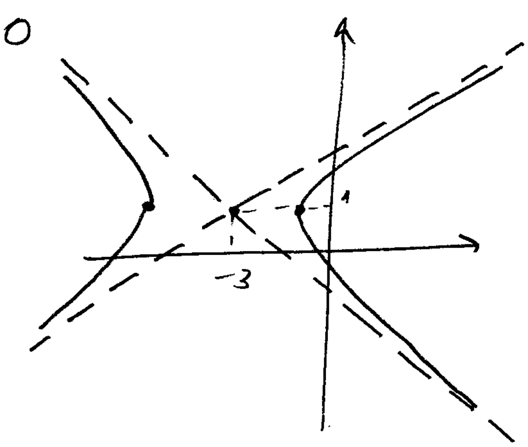
$$(x+3)^2-2(y-1)^2-2=0$$

$$\frac{(x+3)^2}{2}-(y-1)^2=1$$

– hyperbola

Asymptotes:

$$y-1=\pm\frac{x+3}{\sqrt{2}}$$



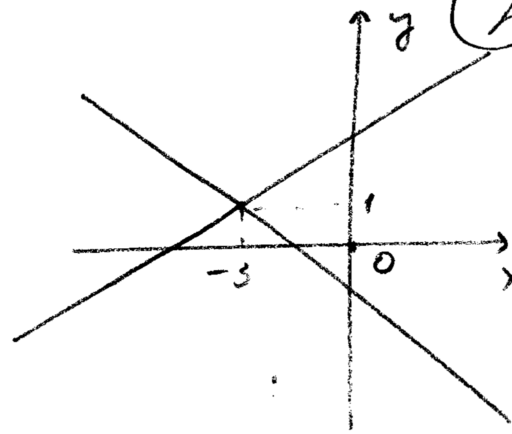
(2)

(e) $(x+3)^2 - 2(y-1)^2 = 0$

$(x+3)^2 = 2(y-1)^2$

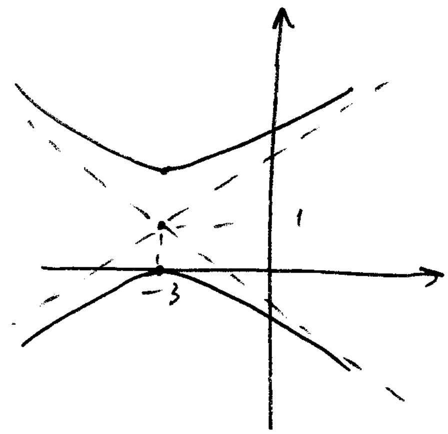
$y-1 = \pm \frac{1}{\sqrt{2}}(x+3)$

– pair of lines



(f) $(x+3)^2 - 2(y-1)^2 + 2 = 0$

$(y-1)^2 - \frac{(x+3)^2}{2} = 1$



2. (a) ellipse

(c) hyperbola

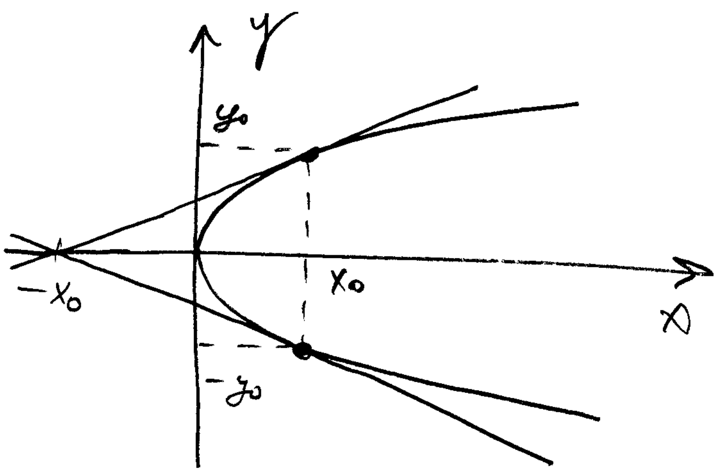
(d) empty set : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = -1$

has no real solutions.

3. $y^2 = 4px$

$$2yy' = 4p$$

$$y' = \frac{2p}{y} \Rightarrow y - y_0 = \frac{2p}{y_0}(x - x_0)$$



$$y_0(y - y_0) = 2p(x - x_0)$$

$$y_0 y = 2p(x - x_0) + y_0^2$$

$$y_0 y = 2p(x - x_0) + 4px_0$$

$$y_0 y = 2p(x + x_0)$$

$$y = 0 \Rightarrow x = -x_0$$

4. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{2x}{a^2} + \frac{2yy'}{b^2} = 0$$

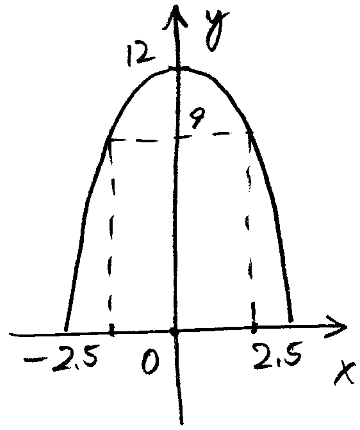
$$y' = -\frac{b^2}{a^2}\frac{x}{y} \Rightarrow y - y_0 = -\frac{b^2}{a^2}\frac{x_0}{y_0}(x - x_0)$$

$$\frac{y_0(y - y_0)}{b^2} + \frac{x_0(x - x_0)}{a^2} = 0$$

$$\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = \frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} = 1$$

$$\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1.$$

5.



$$\frac{x^2}{2.5^2} + \frac{y^2}{12^2} = 1$$

$$y = 9 \Rightarrow \frac{x^2}{2.5^2} + \frac{9^2}{12^2} = 1$$

$$\frac{x^2}{2.5^2} = 1 - \frac{81}{144} = \frac{63}{144}$$

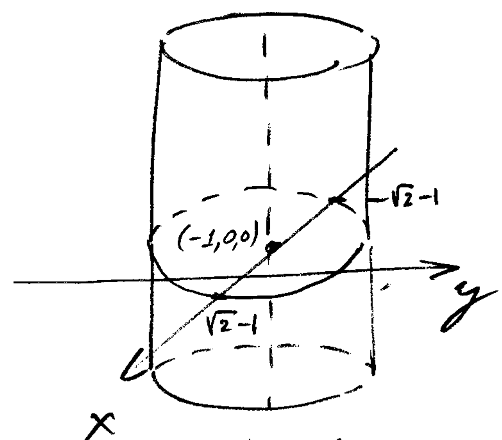
$$x = \pm 2.5\sqrt{\frac{63}{144}} \approx \pm 1.654$$

$$|2x| = 5\sqrt{\frac{63}{144}} \approx \underline{\underline{3.307}} \text{ (ft.)}$$

6. (a) $(x^2+2x+1) + y^2 = 2$

$(x+1)^2 + y^2 = 2$

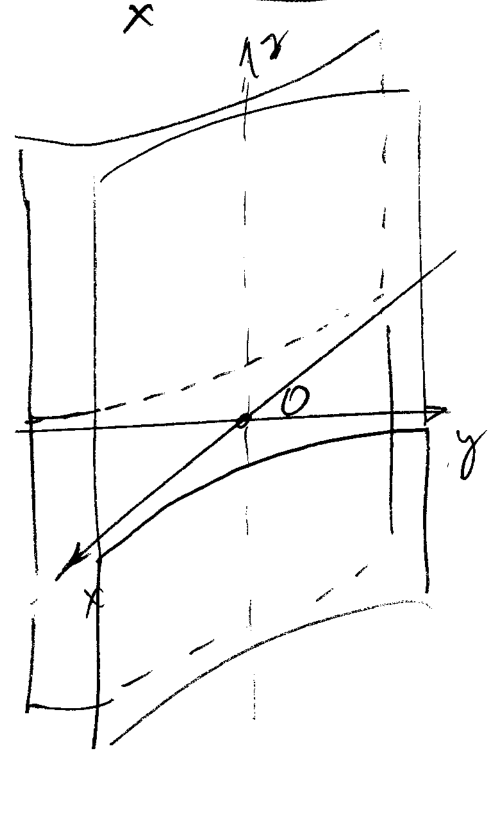
right circular cylinder,
axis $x=-1, y=0$
radius $\sqrt{2}$.



(b) $xy = 1$

– equation of a hyperbola $y = \frac{1}{x}$ on the xy-plane

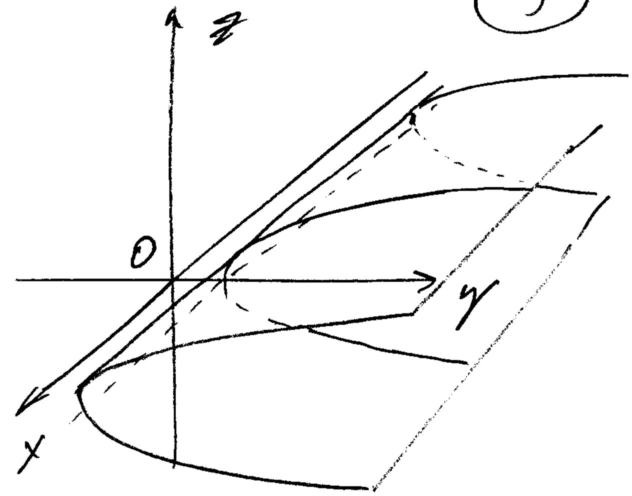
The surface is a cylinder built on this hyperbola:



(c) $y = z^2 + 1$

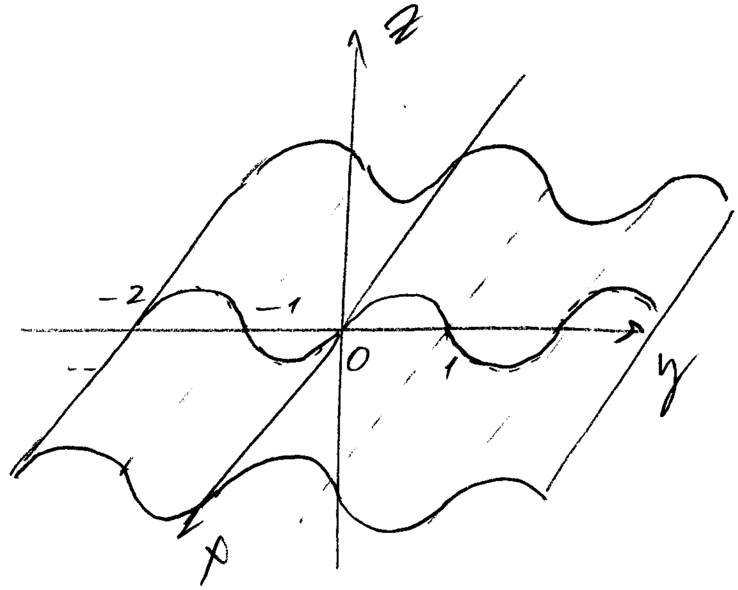
parabola on the yz-plane; rulings parallel to the x-axis

— parabolic cylinder



(d) The curve $z = \sin \pi y$ on the yz-plane; rulings parallel to the x-axis:

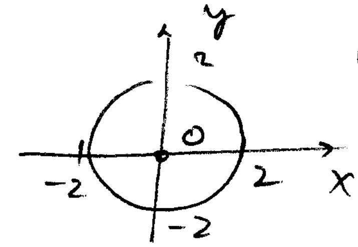
"the sine-shaped plane wave"



7 (a) $x^2+y^2-z^2=0$

$z=2 \Rightarrow x^2+y^2=4$

— circle of radius 2
center @ $(0,0,2)$

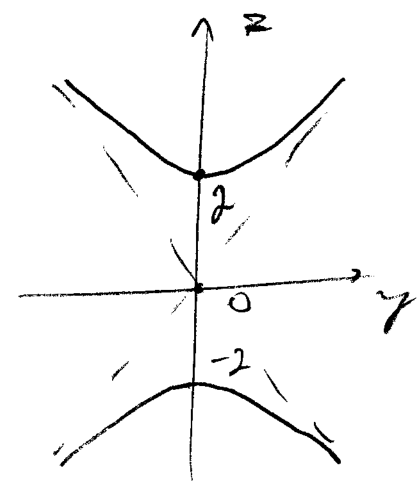


(b) $x^2+y^2-z^2=0$

$x=4 \Rightarrow 4^2+y^2-z^2=0$

$z^2-y^2=4^2$

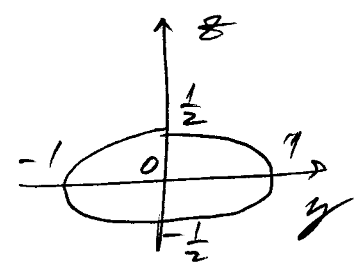
hyperbola with
asymptotes $z=\pm y$



(c) $x^2-2x+y^2+4z^2=0$

$x=1 \Rightarrow y^2+4z^2=1$

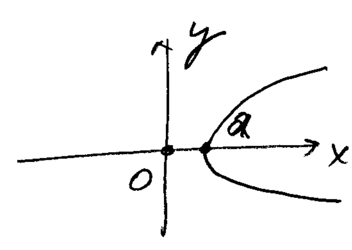
— ellipse with
principal axes of
length 2 (y-dir.)
and 1 (z-dir)



(d) $x-y^2=2$

$y^2=x-2$

parabola,
vertex @ $(2,0,1)$



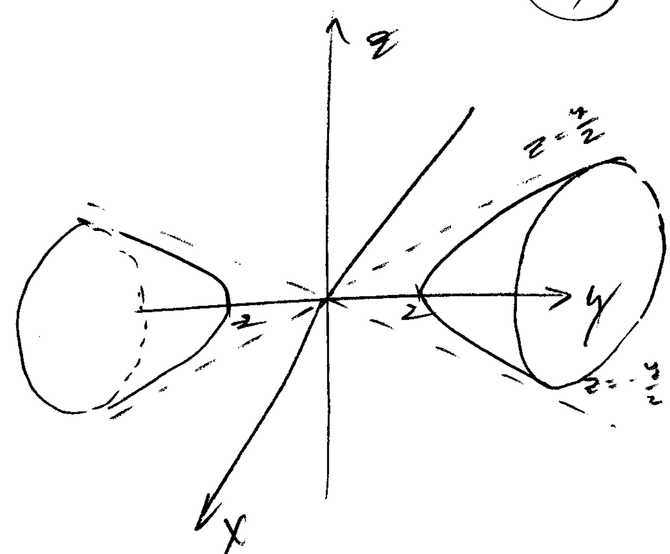
8.(a) $y^2 = x^2 + 4z^2 + 4$

$y^2 - x^2 - 4z^2 = 4$

$\frac{y^2}{4} - \frac{x^2}{4} - z^2 = 1$

hyperboloid of two sheets.

$\frac{x^2}{4} + z^2 = \frac{y^2}{4} - 1$



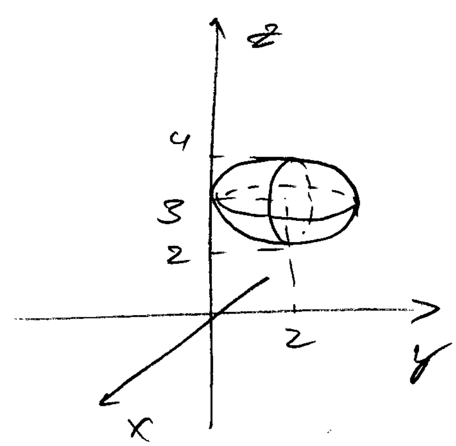
(The surface is "flattened" in the z-direction compared to x and y)

(b) $4x^2 + (y^2 - 4y + 4) - 4 + 4(z^2 - 6z + 9) - 36 + 36 = 0$

$4x^2 + (y-2)^2 + 4(z-3)^2 = 4$

$x^2 + \frac{(y-2)^2}{4} + (z-3)^2 = 1$

– ellipsoid with center at $(0, 2, 3)$

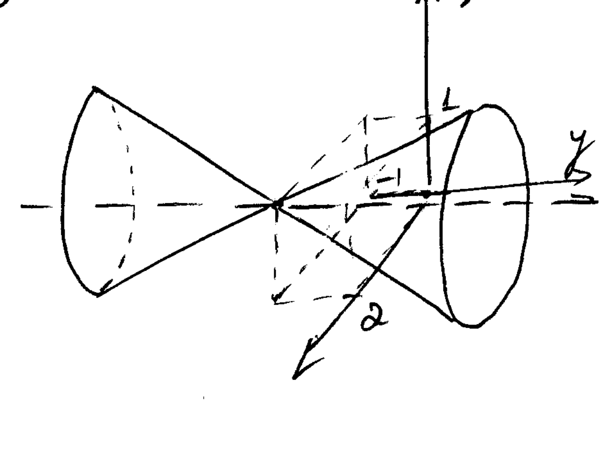


(C) $(x^2 - 4x + 4) - 4 - (y^2 + 2y + 1) + 1 + (z^2 - 2z + 1) - 1 + 4 = 0$

$(x-2)^2 - (y+1)^2 + (z-1)^2 = 0$

Cone with vertex at $(2, -1, 1)$.

y-axis – axis of symmetry.

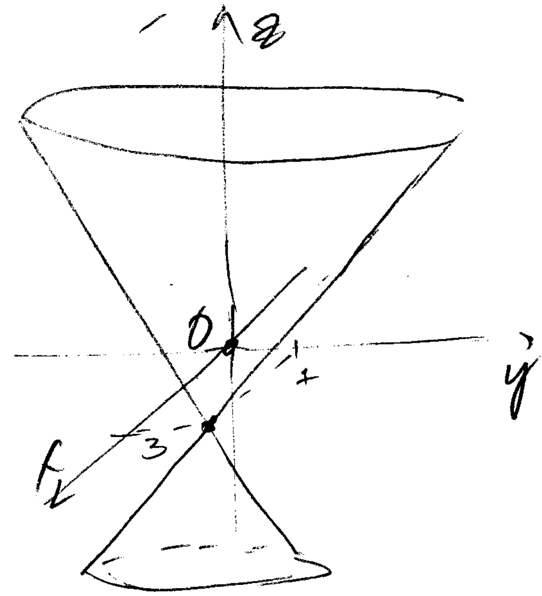


(d) $(x^2+6x+9) - 9 + (y^2-2y+1) - 1 - z^2 + 10 = 0$

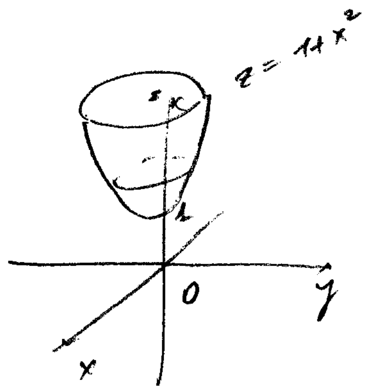
$(x-3)^2 + (y-1)^2 - z^2 = 0$

Circular cone;
vertex $(3, 1, 0)$

z-axis – axis of symmetry



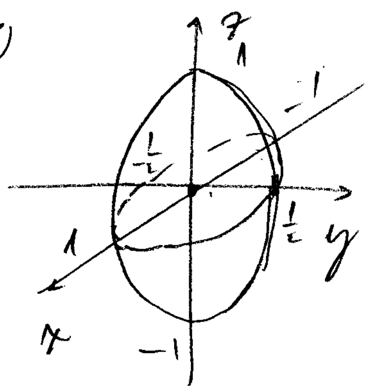
9. (a)



paraboloid

$z = 1 + x^2 + y^2$

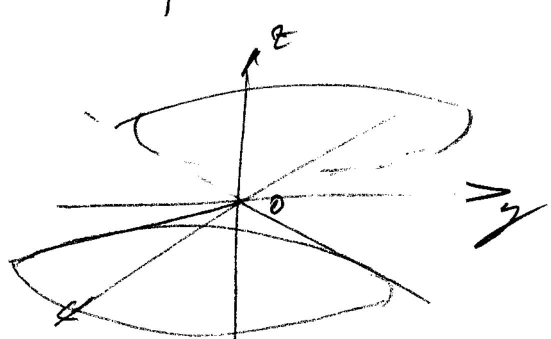
(b)



ellipsoid

$x^2 + 4y^2 + z^2 = 1$

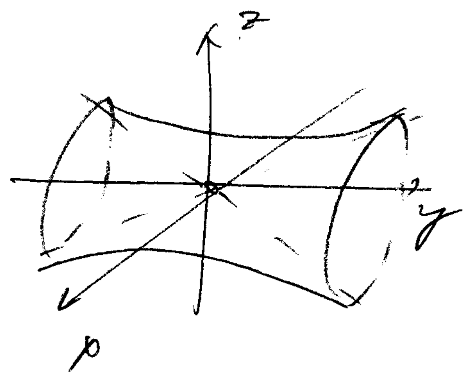
(c)



Cone

$x^2 = 9y^2 + 9z^2$.

(d)



$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

asymptotes:

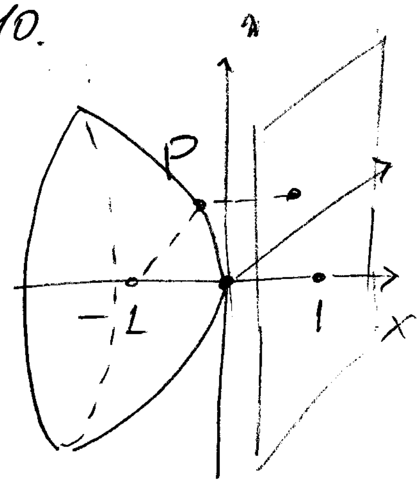
$$\frac{x}{2} = \pm \frac{y}{3}$$

$$y = \pm \frac{3}{2} x$$

hyperboloid of one sheet

$$\frac{x^2}{4} - \frac{y^2}{9} + \frac{z^2}{4} = 1$$

10.



(a) $P(x,y,z)$

$$(x+1)^2 + y^2 + z^2 = (1-x)^2$$

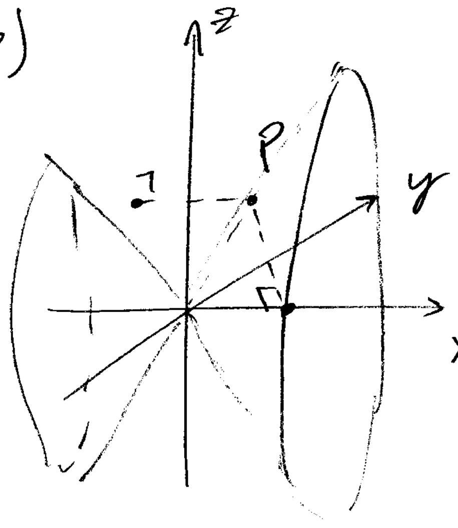
$$\cancel{x^2} + 2x + \cancel{1} + y^2 + z^2 = \cancel{1} - 2x + \cancel{x^2}$$

$$y^2 + z^2 = 4x$$

or $x = \frac{1}{4}(y^2 + z^2)$

— paraboloid of revolution

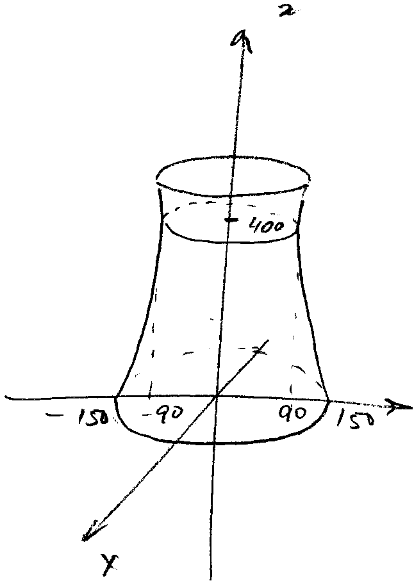
(b)



$$\underbrace{2|x|}_{\text{distance from the } yz\text{-plane}} = \sqrt{y^2 + z^2}$$

$$\underline{4x^2 = y^2 + z^2}$$ — circular cone

11.



(10)

narrowest @ 400 ft

$$\underbrace{\frac{x^2}{a^2}+\frac{y^2}{a^2}}_{\text{circular base}}-\frac{(z-400)^2}{c^2}=1$$

$z=400 \Rightarrow \quad x^2+y^2=90^2$

$\Rightarrow a=90$

$z=0 \quad \Rightarrow \quad x^2+y^2=150^2$

$x^2+y^2=90^2\left(1+\frac{400^2}{c^2}\right)=150^2$

$1+\frac{400^2}{c^2}=\frac{150^2}{90^2}$

$\frac{400^2}{c^2}=\frac{150^2}{90^2}-1$

$c^2=\frac{400^2}{\frac{150^2}{90^2}-1}=\frac{400^2}{\frac{25}{9}-1}$

$=\frac{9}{16}\cdot 400^2$

$=300^2$

$$\boxed{\frac{x^2}{90^2}+\frac{y^2}{90^2}-\frac{(z-400)^2}{300^2}=1}$$