# Homework for Sections 10.5 and 12.6

Due on Thursday, September 25, 2014, 3:30pm.

Directions for graphing the conic section curves: (a) parabola: locate the vertex, the axis of symmetry, determine the stretching factor (identifying a couple of points on the curve is helpful); (b) ellipse: locate the center and the extreme points on the major and minor axes; (c) hyperbola: locate the center, the vertices, and the asymptotes.

1. Classify the equations as representing an ellipse (circle) / hyperbola / parabola / pair of lines, and sketch the curve in the $xy$-plane:

   (a) $x^2 + y^2 + 12y + 27 = 0$

   (b) $2x^2 - x + 2y + \frac{9}{8} = 0$

   (c) $9x^2 - 18x + 4y^2 - 27 = 0$

   (d) $x^2 + 6x - 2y^2 + 4y + 5 = 0$

   (e) $x^2 + 6x - 2y^2 + 4y + 7 = 0$

   (f) $x^2 + 6x - 2y^2 + 4y + 9 = 0$.

2. Determine the type of the curve represented by the equation

$$\frac{x^2}{k} + \frac{y^2}{k-16} = 1$$

   in each of the following cases: (a) $k > 16$; (b) $0 < k < 16$; (c) $k < 0$.

3. Show that the equation of the tangent line to the parabola $y^2 = 4px$ at the point $(x_0, y_0)$ can be written as

$$y_0 y = 2p(x + x_0).$$

   (b) What is the $x$-intercept of this tangent line? Use this fact to draw the tangent line.

4. Show that the equation of the tangent line to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $(x_0, y_0)$ can be written as

$$\frac{xx_0}{a^2} + \frac{yy_0}{b^2} = 1.$$

   Hint: use implicit differentiation; simplify using the equation of the ellipse.

5. A door in the shape of a semi-elliptic arch (half the ellipse) is 12 feet high at the center and 5 feet wide at the base. A rectangular box 9 feet tall is to be slid through the door. What is the largest possible width of the box?

6. Describe and sketch surfaces in $\mathbb{R}^3$ that are represented by the equations

   (a) $x^2 + 2x + y^2 = 1$

   (b) $xy = 1$

   (c) $y = z^2 + 1$

   (d) $z = \sin \pi y$.

7. Find the sections (traces) of the surfaces by the given planes. Sketch the graphs of the corresponding plane curves:

   (a) $x^2 + y^2 - z^2 = 0,\ z = 2$

   (b) $x^2 + y^2 - z^2 = 0,\ x = 4$

   (c) $x^2 - 2x + y^2 + 4z^2 = 0,\ x = 1$

   (d) $x - y^2 - z^2 = 1,\ z = 1$.

8. Reduce the equations to one of the standard forms, classify the surface and sketch it. Use sections by appropriate planes to help the process.

   (a) $y^2 = x^2 + 4z^2 + 4$

   (b) $4x^2 + y^2 + 4z^2 - 4y - 24z + 36 = 0$

   (c) $x^2 - y^2 + z^2 - 4x - 2y - 2z + 4 = 0$

   (d) $x^2 + y^2 - z^2 + 6x - 2y + 10 = 0$.

9. Find an equation for the surface obtained by rotating the given plane curve about the given axis

   (a) The parabola $z = 1 + x^2$; the $z$-axis

   (b) The ellipse $x^2 + 4y^2 = 1$; the $y$-axis

   (c) The straight line $x = 3y$; the $x$-axis

   (d) The hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$; the $y$-axis.

10. (a) Find an equation for the surface consisting of all points $P$ that are equidistant from the point $(-1, 0, 0)$ and the plane $x = 1$. Identify the surface.

    (b) Find an equation for the surface consisting of all points $P$ for which the distance from $P$ to the $x$-axis is twice the distance from the $yz$-plane. Identify the surface.

11. A cooling tower for a nuclear reactor is to be constructed in the shape of a hyperboloid of one sheet (see a photo on page 856 in the textbook). The diameter of the (circular) base is 300 ft, and the minimum diameter, 400 ft above the base is 180 ft. Find an equation for the tower. Use a coordinate system for which the origin is located at the center of the base, and the $x$ and the $y$ axes are in the horizontal plane.

12. Consider the hyperbolic paraboloid

$$z = x^2 - y^2.$$

Introduce new coordinates in the $xy$-plane by setting $X = \frac{1}{\sqrt{2}}(x+y)$, $Y = \frac{1}{\sqrt{2}}(x-y)$. Show that the equation of the paraboloid takes the form

$$z = 2XY$$

in the coordinates $(X, Y, z)$. *[Remark: Coordinates $(X, Y)$ correspond to a rotation of the $(x, y)$-axes through and angle of $45°$. Sketch the axes $X = 0$ and $Y = 0$, and the lines $X = 1$ and $Y = 1$ in the $(x, y)$-plane to see this.]*

13. Consider the hyperbolic paraboloid

$$z = x^2 - y^2.$$

Show that if a point $P(x_0, y_0, z_0)$ is on the paraboloid then there are two lines with parametric equations

$$x = x_0 + at, \quad y = y_0 + bt, \quad z = z_0 + ct,$$

that lie entirely on the paraboloid. *[Remark: Google images under 'Hyperbolic paraboloid as a ruled surface' for an illustration of this fact.]*