Wed, Oct 30, 2013 **MATH 250, Quiz 6** Prof. V. Panferov

Name (print): Solutions.

Each problem is worth 2 points. Show all your work.

1. Find an equation of the tangent plane to the given surface at the specified point:

$$z = 3y^2 - 2x^2 + x, \quad (2, -1, -3).$$

Express your answer in the form $ax + by + cz = d$.

$$f(x,y) = 3y^2 - 2x^2 + x$$

$$f_x = -4x + 1 \; ; \; f_x(2,-1) = -7$$

$$f_y = 6y \; ; \; f_y(2,-1) = -6$$

Tangent plane:

$$z + 3 = -7(x-2) - 6(y+1)$$

$$7x + 6y + z = 5.$$

Please turn over...

2. Use the <u>chain rule</u> to find the derivative dz/dt:

$$z = \sqrt{1 + x^2 + y^2}, \quad x = \ln t, \quad y = \cos t.$$

$; \quad \frac{dx}{dt} = \frac{1}{t}$

$\frac{dy}{dt} = -\sin t$

$$\frac{\partial z}{\partial x} = \frac{2x}{2\sqrt{1+x^2+y^2}} = \frac{x}{\sqrt{1+x^2+y^2}}$$

$$\frac{\partial z}{\partial y} = \frac{2y}{2\sqrt{1+x^2+y^2}} = \frac{y}{\sqrt{1+x^2+y^2}}$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} = \frac{x/t - y\sin t}{\sqrt{1+x^2+y^2}}$$

$$= \frac{\ln t - t\cos t \sin t}{t\sqrt{1+\ln^2 t + \cos^2 t}}.$$

3. Find the maximum rate of change of f at the given point and the direction in which it occurs:

$$f(x, y) = \sin(xy), \quad (x_0, y_0) = (1, 0).$$

$f_x = y\cos(xy) \; ; \quad f_y = x\cos(xy)$

$f_x(1,0) = 0, \quad f_y(1,0) = 1$

$\nabla f = (0, 1)$

$|\nabla f| = 1$ – maximum rate of change

Direction: of the gradient: $(0, 1)$.