Wed, Oct. 16, 2013 **MATH 250, Quiz 5** Prof. V. Panferov

Name (print): Solutions.

Each problem is worth 2 points. Show all your work.

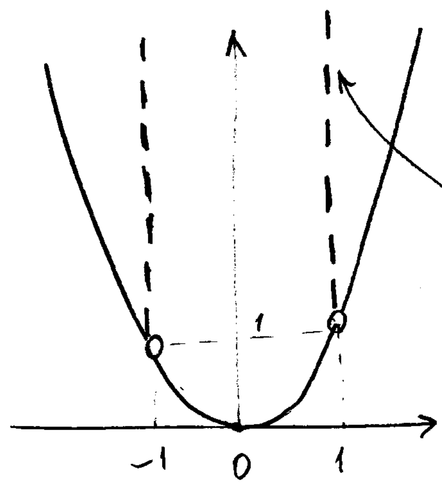
1. Find and sketch the domain D of the function:

$$f(x,y) = \frac{\sqrt{y-x^2}}{1-x^2}$$

$\sqrt{y-x^2}$ is defined when $y - x^2 \geq 0$ $\iff y \leq x^2$.

Division by $1-x^2$ is defined when $1 - x^2 \neq 0$ $\iff x^2 \neq 1$ $\iff x \neq \pm 1$.

$D = \{(x,y) : y \leq x^2,\ x \neq \pm 1\}$



These lines are not included in the domain. (They are part of the boundary.)

2. Draw a contour map of the funcion showing several level curves:

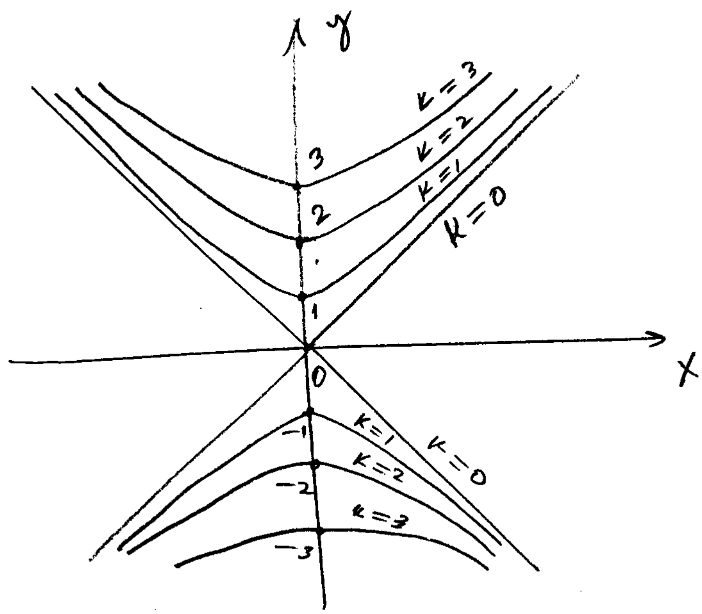
$$f(x,y) = \sqrt{y^2 - x^2}.$$

$\sqrt{y^2-x^2} = k$

$\iff y^2 - x^2 = k^2$

For $k > 0$ this is a hyperbola with $|y| > |x|$ and asymptotes $y = \pm x$.

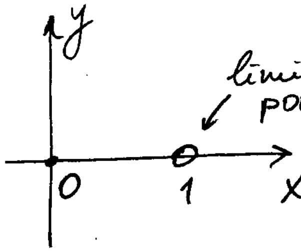
For $k = 0$ this is a pair of lines $y = \pm x$.



Please turn over...

3. Find the limit of the function or show that the limit does not exist:

$$\lim_{(x,y)\to(1,0)} \ln\left(\frac{1+y^2}{x^2+xy}\right).$$



$x^2+xy>0$ near $(1,0)$

$\Rightarrow \ln\left(\frac{1+y^2}{x^2+xy}\right)$ is continuous at $(1,0)$

(a composition of continuous functions.)

$$\lim_{(x,y)\to(1,0)} \ln\frac{1+y^2}{x^2+xy} = \ln\frac{1+0}{1^2+1\cdot 0} = \ln 1 = 0.$$

4. Determine the set of points at which the function is continuous:

$$F(x,y) = \frac{1+x^2+y^2}{1-x^2-y^2}.$$

$F(x,y)$ is a quotient of two functions continuous on all of $\mathbb{R}^2$.

$\Rightarrow F(x,y)$ is continuous for all (x,y) such that $1-x^2-y^2 \neq 0$

(i.e. everywhere in its domain.)

$1-x^2-y^2 \neq 0 \iff x^2+y^2 \neq 1$

$F(x,y)$ is not continuous here