Wed, Oct. 2, 2013 **MATH 250, Quiz 4** Prof. V. Panferov

Name (print): Solutions.

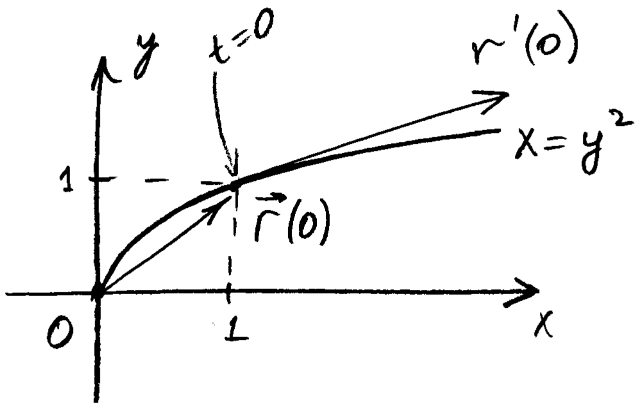
Each problem is worth 2 points. Show all your work.

1. Sketch the curve with the given vector equation. Find $\vec{r}'(t)$. Sketch the vectors $\vec{r}(t)$ and $\vec{r}'(t)$ for the given value of t:

$$\vec{r}(t) = e^{2t}\vec{i} + e^{t}\vec{j}, \quad t = 0.$$

$x = e^{2t}$, $y = e^{t}$ $\Rightarrow$ $x = (e^{t})^2 = y^2$ $\Rightarrow$ the curve lies on the parabola $x = y^2$.

range of $x = e^{2t}$, $y = e^{t}$ is $\{x > 0\}$, $\{y > 0\}$.



$t = 0 \Rightarrow \vec{r}(0) = (1, 1)$.

$\vec{r}'(t) = 2e^{2t}\vec{i} + e^{t}\vec{j}$

$\vec{r}'(0) = (2, 1)$

2. Find the derivative of the vector function $\vec{r}(t) = (t\sin t, t^2, t\cos 2t)$.

$\vec{r}'(t) = (\sin t + t\cos t,\ 2t,\ \cos 2t - 2t\sin 2t)$.

Please turn over...

3. Find the length of the curve:

$$\vec{r}(t) = \sqrt{2}\,t\,\vec{i} + e^t\,\vec{j} + e^{-t}\,\vec{k}, \quad 0 \le t \le 1.$$

$$\vec{r}\,'(t) = \left(\sqrt{2},\ e^t,\ -e^{-t}\right);$$

$$|\vec{r}\,'(t)| = \sqrt{2 + e^{2t} + e^{-2t}}$$

$$= \sqrt{e^{2t} + 2e^t \cdot e^{-t} + e^{-2t}}$$

$$= \sqrt{(e^t + e^{-t})^2} = e^t + e^{-t}.$$

$$\ell(C) = \int_0^1 |\vec{r}\,'(u)|\,du = \int_0^1 e^u + e^{-u}\,du$$

$$= \left[e^u\right]_0^1 + \left[-e^{-u}\right]_0^1 = e - e^{-1} = e - \frac{1}{e}$$

$$\approx 2.3504.$$

4. Find the unit tangent vector $\vec{T}(t)$:

$$\vec{r}(t) = (t, 3\cos t, 3\sin t).$$

$$\vec{r}\,'(t) = \left(1,\ -3\sin t,\ 3\cos t\right)$$

$$|\vec{r}\,'(t)| = \sqrt{1 + 9\sin^2 t + 9\cos^2 t}$$

$$= \sqrt{1 + 9} = \sqrt{10}$$

$$\vec{T}(t) = \left(\frac{1}{\sqrt{10}},\ -\frac{3}{\sqrt{10}}\sin t,\ \frac{3}{\sqrt{10}}\cos t\right)$$