Wed Aug 28, 2013 **MATH 250, Quiz 0** Prof. V. Panferov

Name (print): *Solutions.*

Each problem is worth 2 points. Show all your work.

1. Find the values of x for which the inequality below holds:

$$|x+3| \le |2x-6|.$$

Solution 1:

$-|2x-6| \le x+3 \le |2x-6|$

$|2x-6| = \begin{cases} 2x-6, & x \ge 3 \quad \text{(CASE 1)} \\ 6-2x, & x < 3 \quad \text{(CASE 2)} \end{cases}$

CASE 1: $x \ge 3$

$6-2x \le x+3 \le 2x-6$

$3x \ge 3 \qquad x \ge 9$

$x \ge 1$ — WORKS ALWAYS IN CASE 1

CASE 2: $x \le 3$

$2-6x \le x+3 \le 6-2x$

$x \le 9 \qquad 3x \le 3$

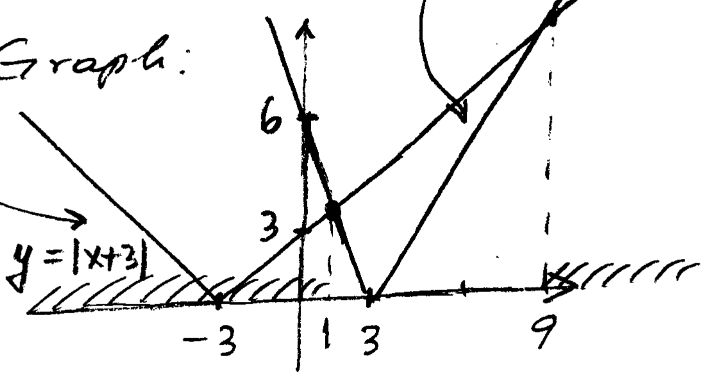
WORKS ALWAYS IN CASE 2 $\qquad x \le 1$

ANSWER: $x \le 1$ OR $x \ge 9$

Solution 2

$|x+3| = \begin{cases} x+3, & x \ge -3 \\ -x-3, & x \le 3 \end{cases}$; $|2x-6| = \begin{cases} 2x-6, & x \ge 3 \\ 6-2x, & x \le 3. \end{cases}$

Graph:



Find points of intersection!

$x+3 = 6-2x$

$\Rightarrow 3x = 3 \Rightarrow x = 1.$

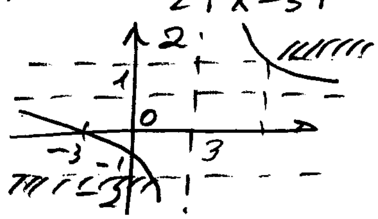
$x+3 = 2x-6$

$\Rightarrow x = 9$

Answer: $x \le 1$ OR $x \ge 9$.

Solution 3: $\left|\dfrac{x+3}{2x-6}\right| \le 1 \iff \dfrac{1}{2}\left|\dfrac{x+3}{x-3}\right| \le 1 \iff \left|\dfrac{x+3}{x-3}\right| \le 2$

Graph the rational function $\dfrac{x+3}{x-3}$:



Solve $\dfrac{x+3}{x-3} = \pm 2$ to find points of intersection.

Please turn over...

2. Find a positive number M such that the absolute value of the given expression does not exceed M if x is in the interval given.

$$f(x) = x^4 - 2x^3 + x^2 - 3x - 5; \quad x \text{ in } [-3,-1].$$

$$|f(x)| \le |x|^4 + 2|x|^3 + |x|^2 + 3|x| + 5$$

$$\le 3^4 + 2\cdot 3^3 + 3^2 + 3\cdot 3 + 5 = 81 + 54 + 9 + 9 + 5$$

$$= 158.$$

Remark: In this case the estimate is the best possible:

$$f(-3) = 158.$$

3. Find a positive number M as in the previous problem

$$\frac{x+2}{x-4}; \quad x \text{ in } [5,8].$$

$|x+2| \le 10$, $|x-4| \ge 1$ for x in $[5,8]$.

$$\Rightarrow \left|\frac{x+2}{x-4}\right| \le \frac{10}{1} = 10.$$

Remark: It can be checked that the function $f(x) = \dfrac{x+2}{x-4}$ is positive and decreasing on $[5,8]$, so the best constant M is

$$f(5) = 7.$$