MATH 250

SECTION 16.2 – SELECTED EVEN PROBLEMS

#4. $$\int_C x\sin y\, ds = \int_a^b x(t)\sin y(t)\, |\vec{r}\,'(t)|\, dt$$

$$\left[\begin{array}{l} C: \ (0,3) \to (4,6) \\ \vec{r}(t) = (0,3)(1-t) + (4,6)t = (4t, 3+3t), \quad 0 \le t \le 1 \\ \vec{r}\,'(t) = (4,-3), \quad |\vec{r}\,'(t)| = \sqrt{4^2+3^2} = 5. \end{array}\right]$$

$$= \int_0^1 4t\sin(3+3t)\cdot 5\, dt = 20\int_0^1 t\sin(3+3t)\, dt$$

$$= 20\int_0^1 t\,\tfrac{1}{3}\big(-\cos(3+3t)\big)'\, dt$$

$$= -\frac{20}{3}\Big[t\cos(3+3t)\Big]_0^1 + \frac{20}{3}\int_0^1 \cos(3+3t)\, dt$$

$$= -\frac{20}{3}\cdot\cos 6 + \frac{20}{3}\left[\frac{1}{3}\sin(3+3t)\right]_0^1$$

$$= \frac{20}{9}\left(-3\cos 6 + \sin 6 - \sin 3\right)$$

#14. $$\int_C y\,dx + z\,dy + x\,dz$$

$C:\quad x = \sqrt{t},\ y = t,\ z = t^2, \qquad 1 \le t \le 4$

$\vec{r}(t) = (\sqrt{t}, t, t^2)$

$\vec{r}\,'(t) = \left(\frac{1}{2\sqrt{t}}, 1, 2t\right) = \left(x'(t), y'(t), z'(t)\right)$

$$\int_C y\,dx + z\,dy + x\,dz = \int_1^4 \left(t \cdot \frac{1}{2\sqrt{t}} + t^2 + \sqrt{t}\,2t\right) dt$$

$$= \int_1^4 \left(t^2 + \frac{1}{2}\sqrt{t} + 2t^{3/2}\right) dt = \left[\frac{t^3}{3}\right]_1^4 + \frac{1}{2}\,\frac{2}{3}\left[t^{3/2}\right]_1^4 + 2\cdot\frac{2}{5}\left[t^{5/2}\right]_1^4$$

$$= \frac{4^3-1}{3} + \frac{2^3-1}{3} + \frac{4}{5}\left(2^5-1\right)$$

$$= \frac{70}{3} + \frac{4\cdot 31}{5} = \frac{350+372}{15} = \frac{722}{15}.$$

#22.

$$\vec{F}(x,y,z) = x\vec{i} + y\vec{j} + xy\,\vec{k}$$

$$\vec{r}(t) = \cos t\,\vec{i} + \sin t\,\vec{j} + t\vec{k}, \quad 0 \le t \le \pi$$

$$\vec{r}\,'(t) = -\sin t\,\vec{i} + \cos t\,\vec{j} + \vec{k}$$

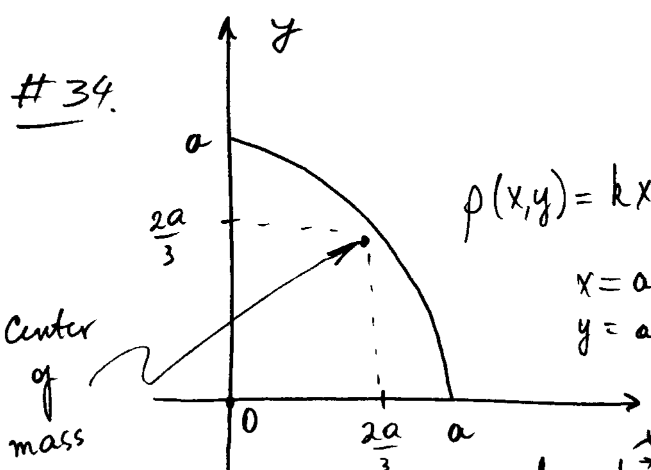
$$\vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t) = -\cos t\sin t + \sin t\cos t + \cos t\sin t$$

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^\pi \cos t\sin t\,dt$$

$$= \int_0^\pi \frac{1}{2}\sin(2t)\,dt = 0$$

since $\int_0^T \sin(\omega t)\,dt = 0$ when $\omega = \frac{2\pi}{T}$.

#34.



$\rho(x,y) = kxy$

$x = a\cos t$

$y = a\sin t$

$ds = |\vec{r}\,'(t)|\,dt = a\cdot dt$

$$m = \int_C \rho(x,y)\,ds$$

$$= \int_0^{\pi/2} ka^2\cos t\sin t\cdot a\,dt$$

$$= ka^3\int_0^{\pi/2} \cos t\sin t\,dt$$

$$= ka^3\int_0^{\pi/2} \cos t\,d(-\cos t)$$

$$= ka^3\left[-\frac{(\cos t)^2}{2}\right]_0^{\pi/2} = \frac{ka^3}{2}$$

Center of mass:

$$x_0 = \frac{1}{m}\int_C x\,\rho(x,y)\,ds$$

$$= \frac{1}{m}\int_0^{\pi/2} ka^3\cos^2 t\,\sin t\,a\,dt$$

$$= \frac{ka^4}{m}\int_0^{\pi/2}\cos^2 t\, d(-\cos t)$$

$$= \frac{ka^4}{m}\left[-\frac{\cos^3 t}{3}\right]_0^{\pi/2} = \frac{1}{3}\frac{ka^4}{m} = \frac{2}{3}a$$

$$\left(m = \frac{ka^3}{2}\right)$$

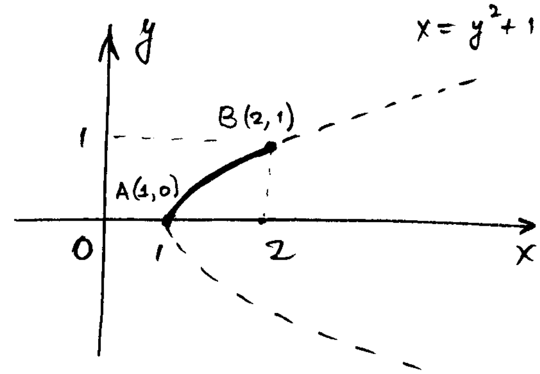
Similarly,

$$y_0 = \frac{1}{m}\int_C y\,\rho(x,y)\,ds$$

$$= \frac{1}{m}\int_0^{\pi/2} ka^3\sin^2 t\,\cos t\,a\,dt.$$

$$= \frac{ka^4}{m}\int_0^{\pi/2}\sin^2 t\,\cos t\,dt = \frac{2}{3}a.$$

\#40. $\vec{F}(x,y) = x^2\vec{i} + ye^x\vec{j}$



parameterize:

C: $\vec{r}(y) = (y^2+1, y)$, $0 \le y \le 1$

$\vec{r}\,'(y) = (2y, 1)$

$$\int_C \vec{F}\cdot d\vec{r} = \int_C P\,dx + Q\,dy = \int_0^1 (y^2+1)\,2y\,dy + ye^{y^2+1}\,dy$$

$$= 2\left[\frac{y^4}{4}\right]_0^1 + 2\left[\frac{y^2}{2}\right]_0^1 + \left[\frac{1}{2}e^{y^2+1}\right]_0^1$$

$$= \frac{1}{2} + 1 + \frac{1}{2}(e^2 - e) = \frac{3}{2} + \frac{e(e-1)}{2}.$$