MATH 250

SECTION 15.7 SELECTED EVEN PROBLEMS

#2.
$$\iiint_E (xy+z^2)\,dx\,dy\,dz$$

$$E = \{(x,y,z):\ 0 \le x \le 2,\ \ 0 \le y \le 1,\ \ 0 \le z \le 3\}$$

$$\begin{pmatrix}\text{Method}\\ \text{I}\end{pmatrix} = \int_0^2\int_0^1\int_0^3 xy+z^2 \quad dz\,dy\,dx$$

$$= \int_0^2\int_0^1\left(3xy + \underbrace{\left[\frac{z^3}{3}\right]_0^3}_{=9}\right) dy\,dx$$

$$= \int_0^2\int_0^1 3xy\,dy\,dx + \underbrace{9\cdot 1\cdot 2}_{=18}$$

$$= \int_0^2 3x\cdot\frac{1}{2}\,dx + 18 = \frac{3}{2}\cdot\left[\frac{x^2}{2}\right]_0^2 + 18$$

$$= \frac{3}{2}\cdot 2 + 18 = 3+18 = 21.$$

$$\begin{pmatrix}\text{Method}\\ \text{II}\end{pmatrix} = \int_0^2\int_0^3\int_0^1 xy+z^2\ dy\,dz\,dx$$

$$= \int_0^2\int_0^3\left(\frac{x}{2} + z^2\right) dz\,dx$$

$$= \int_0^2 \frac{3x}{2} + 9\,dx = \frac{3}{2}\cdot\frac{4}{2} + 9\cdot 2 = 21.$$

$$\begin{pmatrix}\text{Method}\\ \text{III}\end{pmatrix} = \int_0^3\int_0^1\int_0^2 (xy+z^2)\,dx\,dy\,dz$$

$$= \int_0^3\int_0^1 \left(2y + 2z^2\right) dy\,dz$$

$$= \int_0^3 (1 + 2z^2)\,dz = 3 + 2\cdot\frac{3^3}{3} = 21.$$

#4.

$$\int_0^1\int_x^{2x}\int_0^y 2xyz\,dz\,dy\,dx = \int_0^1\int_x^{2x} 2xy\cdot\left[\frac{z^2}{2}\right]_0^y dy\,dx$$

$$= \int_0^1 x\int_x^{2x} y^3\,dy\,dx = \int_0^1 x\,\frac{(2x)^4-x^4}{4}\,dx$$

$$= \int_0^1 4x^5 - \frac{1}{4}x^5\,dx = \frac{15}{4}\int_0^1 x^5\,dx$$

$$= \frac{15}{4}\left[\frac{x^6}{6}\right]_0^1 = \frac{15}{24} = \frac{5}{8}.$$

#10.

$$\iiint_E e^{z/y}\,dx\,dy\,dz$$

$$E = \{(x,y,z):\ 0\le y\le 1,\ y\le x\le 1,\ 0\le z\le xy\}$$

$$= \int_0^1\int_y^1\int_0^{xy} e^{z/y}\,dz\,dx\,dy$$

$$= \int_0^1\int_y^1\left[y\,e^{z/y}\right]_{z=0}^{z=xy} dx\,dy$$

$$= \int_0^1 y\int_y^1 (e^x-1)\,dx\,dy = \int_0^1 y\left[e^x-x\right]_{x=y}^{x=1} dy$$

$$= \int_0^1 y(e^1-1-e^y+y)\,dy$$

$$= (e-1)\int_0^1 y\,dy - \int_0^1 ye^y\,dy + \int_0^1 y^2\,dy$$

$$= \frac{e-1}{2} - \left[ye^y\right]_{y=0}^{y=1} + \int_0^1 e^y\,dy + \frac{1}{3} \qquad \approx 0.1925$$

$$= \frac{e-1}{2} - e + \left[e^y\right]_{y=0}^{y=1} + \frac{1}{3} = \frac{e-1}{2} - 1 + \frac{1}{3} = \frac{e}{2} - \frac{7}{6}$$

#20.

$$\left.\begin{array}{l} y = x^2+z^2 \\ y = 8-x^2-z^2 \end{array}\right\} \Rightarrow$$

intersection:

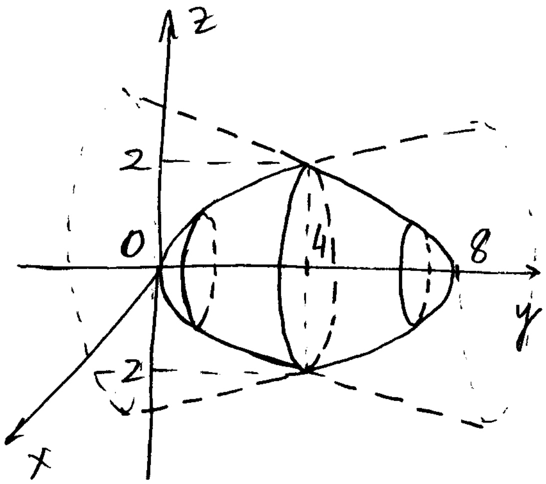
$$x^2+z^2 = 8-x^2-z^2$$

$$2(x^2+z^2)=8$$

$$x^2+z^2=4$$

$$\Rightarrow y=4$$

(the circle of radius 2 in the plane $y=4$.)



$$V = \iiint_E dx\,dy\,dz$$

$$E = \{(x,y,z):\ x^2+z^2 \le 4,\ x^2+z^2 \le y \le 8-x^2-z^2\}$$

$$= \iint_D \int_{x^2+z^2}^{8-x^2-z^2} 1\cdot dy\,dx\,dz$$

$$D = \{(x,z):\ x^2+z^2 \le 4\}$$

$$= \iint_D 8-2(x^2+z^2)\,dx\,dz$$

$$= \int_0^{2\pi}\int_0^2 (8-2r^2)\,r\,dr\,d\theta = 2\pi\int_0^2 (8r-2r^3)\,dr$$

$$= 2\pi\left(8\cdot\left[\frac{r^2}{2}\right]_0^2 - 2\left[\frac{r^4}{4}\right]_0^2\right)$$

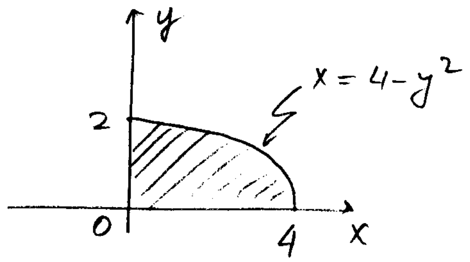
$$= 2\pi(16-8) = 16\pi.$$

#28.
(15.7)

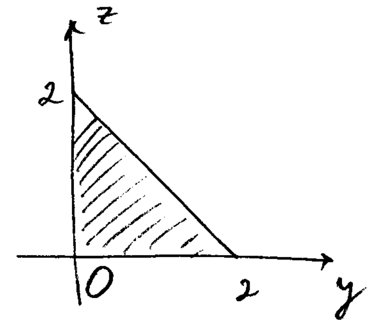
$$\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dx\, dz\, dy$$

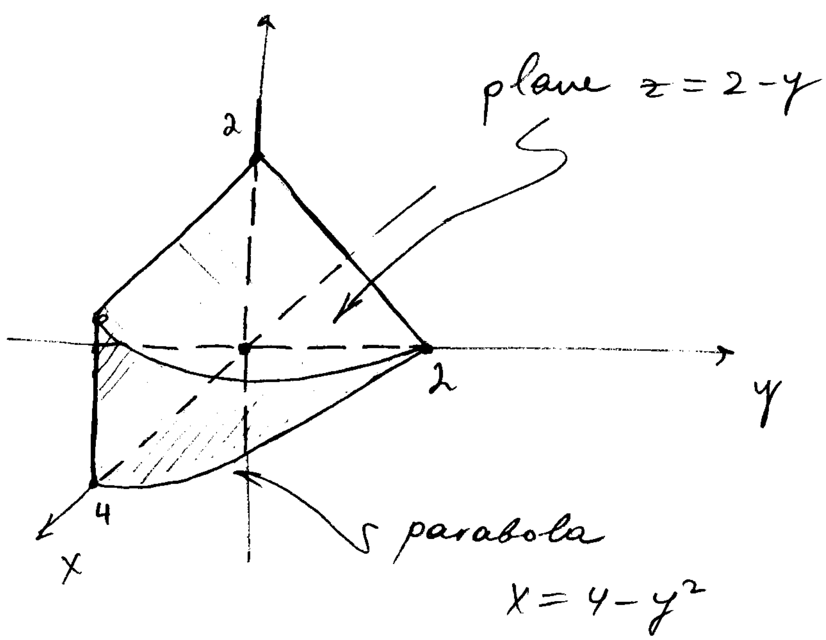
$$E = \{ (x, y, z) : 0 \le y \le 2, \quad 0 \le x \le 4 - y^2, \quad 0 \le z \le 2 - y \}$$

xy - plane:



zy - plane:





Region E is bounded by the parabolic cylinder $x = 4 - y^2$ and the plane $z = 2 - y$, in the first octant.