Turbulence Models

Larry Caretto
Mechanical Engineering 692

Computational Fluid Dynamics

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Outline

- Review last lecture
- Nature of turbulence
- Reynolds-average Navier-Stokes (RNS)
- Mixing length theory
- Models using one differential equation
- Two-equation models, especially k–ε
- Reynolds stress models
- Large-eddy simulation (LES)
- Direct numerical simulation (DNS)

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Review Basic Equations

- Have general equation to use in numerical analysis approaches

$$c\left[\underset{Transient}{\frac{\partial \rho\phi}{\partial t}} + \underset{Convective}{\frac{\partial \rho u_i \varphi}{\partial x_i}}\right] = \underset{Diffusive}{\frac{\partial}{\partial x_i}\Gamma^{(\varphi)}\frac{\partial \varphi}{\partial x_i}} + \underset{"Source"}{S^{(\varphi)}}$$

- Momentum equations separate pressure gradient from other source terms

$S_j^{**} = 0$ for constant μ, ρ and $B_j = 0$

$$\frac{\partial \rho u_j}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_i} = -\frac{\partial P}{\partial x_j} + \frac{\partial}{\partial x_i}\mu\frac{\partial u_j}{\partial x_i} + S_j^{**}$$

$$S_j^{**} = \frac{\partial}{\partial x_i}\mu\frac{\partial u_i}{\partial x_j} + \frac{\partial}{\partial x_j}\left[(\kappa - \frac{2}{3}\mu)\Delta\right] + \rho B_j$$

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Review Solution Approaches

- Solve momentum and continuity for velocity and…
  - Pressure at low Mach numbers where density does not depend on pressure
  - Density at high Mach numbers where pressure is found from equation of state
- Density solution approach

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{F}}{\partial y} + \frac{\partial \mathbf{G}}{\partial z} = \mathbf{H} \qquad \mathbf{U} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho(e + V^2/2) \\ \rho W^{(K)} \end{bmatrix}$$

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Review Fluent Exercise

- Note there are two files case and data
  - Case file has grid information
  - Data file has results
  - Can open and save both at same time
- Will learn about turbulence models this week and numerical algorithms next week
- Use tutorial files as examples of how to use Fluent

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Nature of Turbulence

- Characterized by fluctuations in the flow
- Imperfect ability to model turbulence is major problem in practical applications of CFD
- Turbulent eddies are structures that exist in the flow
  - Largest scale structures get energy from main flow and transfer energy to smaller
  - Smallest scale structures get energy from larger and dissipate energy to viscosity

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Nature of Turbulence II

No smoking in CSUN classrooms, but the smoke from a cigarette shows how laminar flows can transition into turbulent flows and the eddy nature of the turbulent flow structures

Turbulent pipe flow video

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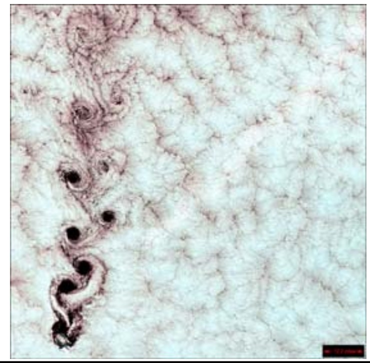
landsat.gsfc.nasa.gov/

- Turbulent flows in clouds from earth satellite shows turbulent structures



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Fluctuation Quantities

- Model flow variables in terms of main flow properties and fluctuations
  - Instantaneous value, ϕ
  - Fluctuation value ϕ'
  - Definition of mean value $\overline{\varphi} = \frac{1}{\Delta t}\int_0^{\Delta t} \varphi dt$
  - Basic result:

$$\varphi = \overline{\varphi} + \varphi'$$

  - Applied to velocity components sometimes use U_i for mean velocity component

$$u_i = \overline{u_i} + u_i' = U_i + u_i'$$

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Turbulent Kinetic Energy

- The kinetic energy per unit mass due to fluctuations in velocity u_i'

$$k = \frac{\overline{u_i' u_i'}}{2} = \frac{\overline{u'u'} + \overline{v'v'} + \overline{w'w'}}{2}$$

- Turbulent intensity = $u'_{rms}/|\mathbf{V}|$

$$u'_{rms} = \sqrt{\frac{1}{3}\left[(u')^2 + (v')^2 + (w')^2\right]/3} = \sqrt{\frac{2k}{3}}$$

$$|\mathbf{V}| = \sqrt{(u)^2 + (v)^2 + (w)^2}$$

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Turbulent Energy Transfer

- Turbulent flows have a series of length scales which transfer energy
  - At largest scales, turbulent structures get kinetic energy from main flow
  - Kinetic energy transfers from larger to smaller length scales
  - Energy is dissipated by viscous effects at the smallest length scales
- Large Reynolds numbers or Grashof numbers mark transition to turbulence

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More on Turbulent Fluctuations

- Definition shows that average fluctuation is zero

$$\varphi = \overline{\varphi} + \varphi' \qquad \overline{\varphi} = \frac{1}{\Delta t}\int_0^{\Delta t} \varphi dt$$

$$\overline{\varphi'} = \frac{1}{\Delta t}\int_0^{\Delta t} (\varphi - \overline{\varphi}) dt = \frac{1}{\Delta t}\int_0^{\Delta t} \varphi dt - \frac{1}{\Delta t}\int_0^{\Delta t} \overline{\varphi} dt = \overline{\varphi} - \overline{\varphi} = 0$$

- The mean value is a constant and the average of a constant is just that constant

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Average of a Product

- The mean of the product of two flow properties ϕ and ψ, written as $\overline{\psi\varphi}$, is the sum of two terms:
  - The product of the means of each individual term $\overline{\varphi}$ and $\overline{\psi}$
  - The mean of the product of the two fluctuation quantities $\overline{\varphi'\psi'}$ (correlation term)

$$\overline{\psi\varphi} = \overline{\varphi}\,\overline{\psi} + \overline{\varphi'\psi'}$$

  - Although $\overline{\varphi'}$ and $\overline{\psi'}$ are zero $\overline{\varphi'\psi'}$ is not zero
  - Derivation on next slide

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Average of a Product

- Two flow properties ϕ and ψ

$$\overline{\psi\varphi} = \frac{1}{\Delta t}\int_0^{\Delta t}(\overline{\varphi}-\varphi')(\overline{\psi}-\psi')dt = \frac{1}{\Delta t}\int_0^{\Delta t}\overline{\varphi}\,\overline{\psi}dt - \frac{1}{\Delta t}\int_0^{\Delta t}\overline{\varphi}\psi' dt$$

$$-\frac{1}{\Delta t}\int_0^{\Delta t}\varphi'\overline{\psi}dt + \frac{1}{\Delta t}\int_0^{\Delta t}\varphi'\psi' dt = \overline{\varphi}\,\overline{\psi}\frac{1}{\Delta t}\int_0^{\Delta t}dt - \overline{\varphi}\frac{1}{\Delta t}\int_0^{\Delta t}\psi' dt$$

$$-\overline{\psi}\frac{1}{\Delta t}\int_0^{\Delta t}\varphi' dt + \frac{1}{\Delta t}\int_0^{\Delta t}\varphi'\psi' dt = \overline{\varphi}\,\overline{\psi} - 0 - 0 + \frac{1}{\Delta t}\int_0^{\Delta t}\varphi'\psi' dt$$

$$\overline{\psi\varphi} = \overline{\varphi}\,\overline{\psi} + \overline{\varphi'\psi'}$$

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More on Averages

- Root-mean square properties based on turbulent fluctuations

$$\varphi_{rms} = \sqrt{\overline{(\varphi')^2}} = \sqrt{\frac{1}{\Delta t}\int_0^{\Delta t}(\varphi')^2 dt}$$

- Average of a space derivative is space derivative of average

$$\overline{\frac{\partial\varphi}{\partial x_i}} = \frac{1}{\Delta t}\int_0^{\Delta t}\frac{\partial\varphi}{\partial x_i}dt = \frac{\partial}{\partial x_i}\left[\frac{1}{\Delta t}\int_0^{\Delta t}\varphi dt\right] = \frac{\partial\overline{\varphi}}{\partial x_i}$$

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Reynolds-Average

- Reynolds average transport equation (including Navier Stokes, called RANS)
  - Start with general transport equation

$$c\left[\frac{\partial\rho\phi}{\partial t} + \frac{\partial\rho u_i\varphi}{\partial x_i}\right] = \frac{\partial}{\partial x_i}\Gamma^{(\varphi)}\frac{\partial\varphi}{\partial x_i} + S^{(\varphi)}$$

  - Look at steady-state, zero-source, constant properties

$$\frac{\partial u_i\varphi}{\partial x_i} = \gamma^{(\varphi)}\frac{\partial}{\partial x_i}\frac{\partial\varphi}{\partial x_i} \qquad \gamma^{(\phi)} = \frac{\Gamma^{(\phi)}}{\rho c}$$

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What is $\gamma^{(\phi)} = \Gamma^{(\phi)}/\rho c$?

- Recall definition of $\Gamma^{(\phi)}$ as general transport coefficient

ϕ	u	v	w	e	h	T	T
$\Gamma^{(\phi)}$	μ	μ	μ	k/c_v	k/c_p	k	k
c	1	1	1	1	1	c_v	c_p
$\gamma^{(\phi)}$	μ/ρ	μ/ρ	μ/ρ	$k/\rho c_v$	$k/\rho c_p$	$k/\rho c_v$	$k/\rho c_p$

- Dimensions for $\gamma^{(\phi)}$ are L^2/T
  - Kinematic viscosity, $\nu = \mu/\rho$
  - Thermal diffusivity $\alpha = k/\rho c_p$

k is thermal conductivity

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Reynolds Average II

- Take time average of last equation

$$\frac{1}{\Delta t}\int_0^{\Delta t}\left[\frac{\partial u_i\varphi}{\partial x_i}\right]dt = \frac{1}{\Delta t}\int_0^{\Delta t}\left[\gamma^{(\varphi)}\frac{\partial}{\partial x_i}\frac{\partial\varphi}{\partial x_i}\right]dt$$

- Time average of derivatives are derivatives of time average

$$\frac{\partial\overline{u_i\varphi}}{\partial x_i} = \gamma^{(\varphi)}\frac{\partial}{\partial x_i}\frac{\partial\overline{\varphi}}{\partial x_i}$$

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Reynolds Average III

- Use expression for average of a product to compute average of $u_i\phi$

$$\frac{\partial \overline{u_i \varphi}}{\partial x_i} = \frac{\partial\left(\overline{u_i}\,\overline{\varphi} + \overline{u_i'\varphi'}\right)}{\partial x_i} = \gamma^{(\varphi)}\frac{\partial}{\partial x_i}\frac{\partial \overline{\varphi}}{\partial x_i}$$

- Compare to equation before averaging
  - Overall values replaced by averages
  - Add a new term: the average of the product of two fluctuations
- Have to compute this product term

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Reynolds Average IV

- Combine fluctuation product term with diffusive flux

$$\frac{\partial \overline{u_i}\,\overline{\varphi}}{\partial x_i} = \frac{\partial}{\partial x_i}\left[\gamma^{(\varphi)}\frac{\partial \overline{\varphi}}{\partial x_i} - \overline{u_i'\varphi'}\right]$$

- Boussinesq approximation: the turbulent fluctuation is proportional to the gradient of the mean property with empirical turbulent transport coefficient, $\gamma_t^{(\phi)}$

$$\overline{u_i'\varphi'} = -\gamma_t^{(\varphi)}\frac{\partial \overline{\varphi}}{\partial x_i}$$

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Reynolds Average IV

- Combine fluctuation product term with diffusive flux

$$\frac{\partial \overline{u_i}\,\overline{\varphi}}{\partial x_i} = \frac{\partial}{\partial x_i}\left[\gamma_l^{(\varphi)}\frac{\partial \overline{\varphi}}{\partial x_i} + \gamma_t^{(\varphi)}\frac{\partial \overline{\varphi}}{\partial x_i}\right] = \frac{\partial}{\partial x_i}\left[\left(\gamma_l^{(\varphi)} + \gamma_t^{(\varphi)}\right)\frac{\partial \overline{\varphi}}{\partial x_i}\right]$$

- Turbulent transport coefficient, $\gamma_t^{(\phi)}$ now contains our ignorance about turbulence
- Seek ways to model this term
- Usually much larger than laminar term so that latter is often neglected

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Momentum Terms

- Apply general formulation to momentum equation where $\phi = u_j$ in j direction
  - For momentum equations, $\gamma^{(\phi)} = \nu$
- Also have time averaged pressure gradient $\partial p/\partial x_j$ which is $\partial \overline{p}/\partial x_j$
- Time average of momentum equation gives

$$\frac{\partial \overline{u_i}\,\overline{u_j}}{\partial x_i} + \frac{\partial \overline{u_i'u_j'}}{\partial x_i} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_j} + \frac{\partial}{\partial x_i}\left[\nu\frac{\partial \overline{u_j}}{\partial x_i} - \overline{u_i'u_j'}\right]$$

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Momentum Terms II

- Move fluctuation product to right side

$$\frac{\partial \overline{u}_i\overline{u}_j}{\partial x_i} = -\frac{\partial \overline{p}}{\partial x_j} + \frac{\partial}{\partial x_i}\left(\nu\frac{\partial \overline{u}_j}{\partial x_i} - \frac{\rho\overline{u_i'u_j'}}{\rho}\right)$$

- Resulting terms are called the Reynolds stress terms
- Nine such terms but only six are unique
- Simple models look at only one term
- Advanced models try to compute all six

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Turbulent Viscosity

- Arguments based on dimensional analysis
- Define characteristic velocity scale, $\mathfrak{v}$, and length scale, ℓ
- Consider kinematic viscosity, ν, with dimensions of L^2/T
- Dimensions of $\mathfrak{v}$ and ℓ and L/T and L
- Dimensionally correct choice for n is product of $\mathfrak{v}$ and ℓ or $\nu_T = C\,\mathfrak{v}\,\ell$

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Turbulence Modeling

- Reynolds averaging terms like $\rho u'\phi'$ modeled by a turbulent transport coefficient $\gamma_t^{(\phi)}$, *e.g.*, turbulent viscosity ν
- To use this approach we have to find ways to compute ν and the general $\gamma_t^{(\phi)}$
- Various turbulence models proposed
  - Some use simple concepts
  - Others require numerical solution of one or more partial differential equations (PDE)
    - PDEs have same form as other CFD equations

Mixing Length Theory

- Originally proposed by Prandtl in 1925
- Basic idea
  - Consider a turbulent fluid with a mean temperature gradient
  - A fluid particle that moves from the cold region to the hot region will take on the characteristics of the hot region after it moves through one mixing length, ℓ
  - The temperature fluctuations are related to the mean gradient as $T' = -\ell \dfrac{\partial \overline{T}}{\partial y}$

Mixing Length

- By dimensional arguments $\nu_t = \mu_t/\rho = C\mathfrak{v}\ell$, where C is dimensionless constant
- For simple flows have only one important Reynolds stress, $-\rho u'v'$
- If length scale is measure of largest eddy size, one possible dimensionally correct velocity scale is $\mathfrak{v} = c\ell|\partial U/\partial y|$
  - c is dimensionless constant which is different from constant C in $\nu_t = C\mathfrak{v}\ell$

Mixing Length II

- If $\nu_t = \mu_t/\rho = C\mathfrak{v}\ell$, and $\mathfrak{v} = c\ell|\partial U/\partial y|$, then $\nu_t = Cc\ell|\partial U/\partial y|\ell = K\ell^2|\partial U/\partial y|$
- This gives the Reynolds stress as follows $$-\rho\overline{v'u'} = -\rho\ell^2\left|\frac{\partial \overline{u}}{\partial y}\right|\frac{\partial \overline{u}}{\partial y}$$
- Other flow properties use turbulent transport ratio, $\sigma_t = \mu_t/\Gamma_t^{(\phi)}$

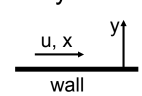
$$-\rho\overline{v'\phi'} = -\frac{1}{\sigma_t}\rho\ell^2\left|\frac{\partial \overline{u}}{\partial y}\right|\frac{\partial \overline{\phi}}{\partial y}$$

Boundary Layers

- Turbulent flow next to a wall has laminar sublayer and transition to fully turbulent flow
- Experimental data and simplified analysis give empirical equations for velocity profile of turbulent boundary layer
- Simplified theory used for turbulent "wall functions" to give boundary conditions for turbulent flows in CFD

Boundary Layer Profile

- Flow in x direction (velocity u) with y as distance perpendicular to wall
  - Wall shear stress = τ_w
  - u_τ is called friction velocity



- Define dimensionless variables
  - y^+ is dimensionless distance from wall
  - u^+ is dimensionless velocity parallel to wall

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}} \qquad u^+ = \frac{u}{u_\tau} \qquad y^+ = \frac{y}{y_\tau} = \frac{u_\tau y}{\nu}$$

Boundary Layers II

- Law of the wall: $u^+ = f(y^+)$
- No equation $5 < y^+ < 30$
- In other regions

$$u^+ = \begin{cases} y^+ & y^+ < 5 \\ \dfrac{\ln(y^+)}{\kappa} + B = \dfrac{\ln(Ey^+)}{\kappa} & 30 < y^+ < 500 \\ u^+_{\max} - \dfrac{1}{\kappa}\ln\left(\dfrac{y}{\delta}\right) - A & y^+ > 500 \end{cases}$$

- B = 5.5; E = 9.8, κ = 0.41

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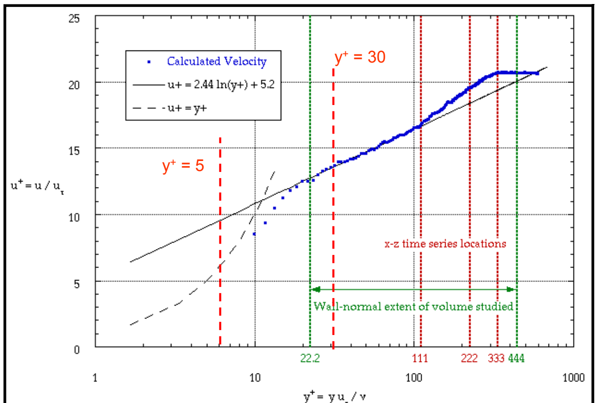

C. Delo and A.J. Smits, Volumetric visualization of coherent structure in a low Reynolds number turbulent boundary layer, *International Journal of Fluid Dynamics (1997), Vol. 1, Article 3*, Figure 6, downloaded 1/31/2010 from http://www.eng.nus.edu.sg/ijfd/1997_vol1/paper3/figures/Figure6.html

Spalart-Allmaras Model

- Solves one PDE for $\tilde{\nu}$ which is used to find ν_t from equation at right
  - Default value: C_{v1} = 7.1
  - In most of flow away from walls $\nu_t = \tilde{\nu}$

$$\nu_t = \frac{\tilde{\nu}\left(\dfrac{\tilde{\nu}}{\nu}\right)^3}{\left(\dfrac{\tilde{\nu}}{\nu}\right)^3 + C_{v1}^3}$$

- Developed for aerospace applications to external flows with walls
- Has recently been applied to other flow situations

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k-epsilon (k-ε) Model

- Most popular model of turbulence
  - Original and modified versions
- Works well in confined flows with no rotating parts
- Less applicable in external flows and oscillating flows
- Requires solution of two PDEs one for turbulent kinetic energy, k, and one for turbulent dissipation, ε

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What is ε?

- Dissipation rate, ε, is rate of kinetic energy transfer from smallest eddies working against the viscous forces
- Defined in terms of deformation rates, e_{ij}

$$e_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \qquad \tau_{ij} = \mu e_{ij} + (\kappa - \frac{2}{3}\mu)\Delta\delta_{ij}$$

- Definition of ε is $\varepsilon = 2\nu\overline{e'_{ij}e'_{ij}}$
- Dimensions of ε are energy divided by [(mass)(time)], e.g. m^2/s^3

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Computing ν_t from k and ε

- Back to basic equation that ν_t is the product of a length scale and a velocity scale: $\nu_t = \mu_t/\rho = C_\mu \mathfrak{v} \ell$,
- Use $\mathfrak{v} = k^{1/2}$ as velocity scale
- Length scale, $\ell = k^{3/2}/\varepsilon$
- Result for viscosity is $\nu_t = \mu_t/\rho = C_\mu k^2/\varepsilon$
- Solve PDEs for k and ε
- Can derive form of equation for k

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k Equation

- Can derive following balance equation

$$\frac{\partial \rho k}{\partial t}+\frac{\partial \rho \overline{u_i} k}{\partial x_i}=\frac{\partial}{\partial x_i}\mu\frac{\partial k}{\partial x_i}-\frac{\partial}{\partial x_i}\left(\frac{\rho}{2}\overline{u_i'u_j'u_j'}+\overline{p'u_i'}\right)+\rho\overline{u_i'u_j'}\frac{\partial \overline{u_i}}{\partial x_j}-\rho\varepsilon$$

- Can identify transient, convection and diffusion terms; remainder is source
- Terms with three fluctuating components cannot be computed without introducing terms with four fluctuating components

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Modeling k Equation Terms

- Turbulent diffusion of kinetic energy

$$-\left(\frac{\rho}{2}\overline{u_i'u_j'u_j'}+\overline{p'u_i'}\right)\approx\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_i}$$

- Empirical constant: σ_k
- Production term, P_k , uses model for Reynolds stresses

$$P_k=\rho\overline{u_i'u_j'}\frac{\partial \overline{u_i}}{\partial x_j}=-\mu_t\left(\frac{\partial \overline{u_i}}{\partial x_j}+\frac{\partial \overline{u_j}}{\partial x_i}\right)\frac{\partial \overline{u_i}}{\partial x_j}$$

- Implied summation over two indices

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The k Equation

- Final form with modeled terms

$$\frac{\partial \rho k}{\partial t}+\frac{\partial \rho \overline{u_i} k}{\partial x_i}=\frac{\partial}{\partial x_i}\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_i}+P_k-\rho\varepsilon$$

- Usually ignore laminar viscosity

$$\frac{\partial \rho k}{\partial t}+\frac{\partial \rho \overline{u_i} k}{\partial x_i}=\frac{\partial}{\partial x_i}\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_i}+P_k-\rho\varepsilon$$

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The ε Equation

- Derivation similar to that for k
  - Obtain partial differential equation with terms that have to be modeled

$$\frac{\partial \rho \varepsilon}{\partial t}+\frac{\partial \rho \overline{u_i}\varepsilon}{\partial x_i}=\frac{\partial}{\partial x_i}\frac{\mu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_i}+C_{\varepsilon 1}\frac{\varepsilon}{k}P_k-\rho C_{\varepsilon 2}\frac{\varepsilon^2}{k}$$

- Empirical constants in k-ε model

$$C_\mu=0.09 \quad C_{\varepsilon 1}=1.44 \quad C_{\varepsilon 2}=1.92 \quad \sigma_k=1.0 \quad \sigma_\varepsilon=1.3$$

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Boundary Conditions

- Must specify values of k and ε at all boundaries (walls, inlets, outlets, etc.)
- Solid walls use “wall functions” to provide correct result at first node
- Based on boundary layer results for flat walls
  - u is velocity parallel to wall
  - y is direction normal to wall

$$u^+=\frac{u}{u_\tau}=\frac{\ln(Ey^+)}{\kappa}=\frac{1}{\kappa}\ln\left(\frac{Eyu_\tau}{\nu}\right)$$

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What is u_τ?

- Friction velocity, $u_\tau = (\tau_w/\rho)^{1/2}$
- Need a value for τ_w
- Assume production and dissipation of turbulence are nearly equal near wall
- Derive following equation: $\tau_w = (C_\mu k^2)^{1/4}$
- Combine with u^+ equation to get result for u at first node in from the wall

$$u=u_\tau\ln\left(\frac{Eyu_\tau}{\nu}\right)=C_\mu^{1/4}\sqrt{k}\ln\left(\frac{EyC_\mu^{1/4}\sqrt{k}}{\nu}\right)$$

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Inlet and Outlet Conditions

- Ideally have data on similar flows that relate k and ε to inlet properties
- Failing that use the following equations

$$k = Cu_{inlet}^2 \qquad \ell = 0.07L \qquad \varepsilon = C_\mu^{3/4}\frac{k}{\ell}$$

- C, proportional to square of turbulence intensity, typically about 0.01 to 0.05
- L is a characteristic length of the inlet (e.g., the hydraulic diameter)
- Use zero gradient conditions for outlet
- k = ε = 0 for free stream

Low Reynolds Numbers

- Special wall functions are required for “low” Reynolds numbers flows
- Define wall damping functions, f
  - Use f_μ to compute $\mu_t = C_\mu f_\mu k^2/\varepsilon$
  - Use f_1 and f_2 to modify production and dissipation terms in ε equation
  - Use laminar viscosity in addition to turbulent viscosity in both k and ε equations
  - Different forms for damping functions

Reynolds Stress Models

- Solve a differential equation for each unique Reynolds stress
  - PDEs for u’u’, u’v’, u’w’, v’v’, v’w’ w’w’
  - Find k = (u’u’ + v’v’ + w’w’)/2
  - Also solve ε equation
- Should work well for strongly anisotropic flows, but modeling assumptions may limit accuracy of model
- Usually applied to flows with rotating flow or swirl

Reynolds Stress Models II

- Other transport equations compute turbulent viscosity, ν_t, and use Prandtl number, σ_ϕ, to get $\gamma^{(\phi)} = \nu_t / \sigma_\phi$
- Algebraic Reynolds stress model is simplification that solves k and ε equations then gets Reynolds stress by six simultaneous algebraic equations
  - Equations for u’u’, u’v’, u’w’, v’v’, v’w’ w’w’
  - Other turbulence quantities: $\gamma^{(\phi)} = \nu_t / \sigma_\phi$

Large Eddy Simulation (LES)

- Use small grid scale to compute largest eddies as part of the calculation
- Use sub grid scale models to get results for the finest turbulence scales
- Now available in production codes
- Generally not worth the extra computational cost except for complex flows

Direct Numerical Simulation (DNS)

- Use a grid fine enough to resolve the smallest turbulence structures which are 10^{-4} to 10^{-5} m
- A volume of 30 m^3 for flow around a motorcycle would require 3×10^{12} nodes
- DNS not practical for engineering calculations, but an important research tool for examining turbulence properties and testing other turbulence models

Model Guidance

- Use model which has been used previously for your problem
  - Previous work at your organization or from literature research
- Consult user's manual for CFD code regarding increased computer time and memory use for more complex models
- Use default constants in model unless you have specific data to justify alternative values

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Model Guidance II

- Review material on turbulence models to see if they can handle unusual features of the flow you are modeling
  - Low Reynolds number (non-equilibrium) turbulence
  - High strain rates
  - Adverse pressure gradients
  - Rotating machinery
  - Compressible flows
  - Other complex flows

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Model Guidance III

- Whatever model you use, make sure that you have proper boundary conditions
  - Use special wall functions for non-equilibrium turbulence when laminar sublayer is not resolved
  - Use correct grid spacing for first node from wall for choice of wall functions or resolving laminar sublayer
    - Check this after calculations

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Conclusions

- k-ε most common turbulence model for non-aerospace engineering applications
  - widely regarded as having many shortcomings in representing turbulence
  - most widely validated model
  - probably best choice for applications without strong directional effects or rotational flows
  - Renormalizable group and realizable k-ε models can give better results

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Conclusions II

- Spalart-Allmaras model developed especially for aerospace applications with wall-bounded flows
  - One equation model
- Relatively new model now seeing applications in areas other than its initial aerospace applications
  - Adverse pressure gradients
  - Turbomachinery

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Conclusions III

- Reynolds stress model has had success for flows with directional effects and rotational flows
  - Requires solution of seven partial differential equations to compute turbulent viscosity
  - Algebraic version has been used
- Other models available which may have better accuracy for limited range of flows

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Conclusions IV

- LES used for complex flows particularly transient and oscillating flows
- Not usually required for common engineering problems
- Choice of turbulence model should be based on previous success of model in similar applications
- No one right model to choose

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