Introduction to Fluid Statics and Manometers

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Mechanical Engineering 390

Fluid Mechanics

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California State University Northridge

Outline

- Review course introduction
- Pressure independent of direction
- Pressure-density-distance relationship in a static (nonmoving fluid)
- Use of manometers for pressure measurements
- Calculations with manometers.

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Review

- Dimensions and units
  - SI, BG and EE unit systems
- Fluid density, ρ, (mass/volume) and specific weight (weight/volume), $\gamma = \rho g$, and specific gravity
- States of matter and vapor pressure
- Viscosity
- Surface tension

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Review Typical Units

Quantity	**SI units**	**EE units**	**BG units**
Density	kg/m^3	lb_m/ft^3	$slug/ft^3$
Pressure & shear stress	kPa = kN/m^2	1 psi = 1 lb_f/in^2 = 144 psf = 144 lb_f/ft^2	
Velocity	m/s	ft/s	
Viscosity	$N{\cdot}s/m^2$ = $kg/m{\cdot}s$	$lb_f{\cdot}s/ft^2$ = 32.2 $lb_m/ft{\cdot}s$	$lb_f{\cdot}s/ft^2$ = $slug/ft{\cdot}s$
Specific weight = ρg	N/m^3	lb_f/ft^3	
	Tabulated values at standard gravity		

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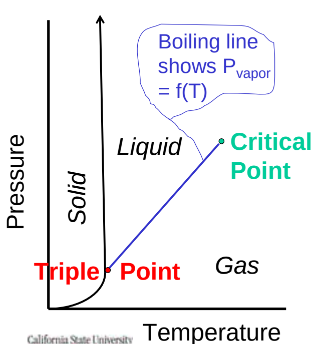
Review States of Matter



- **Triple point:** solid, liquid and vapor coexist
- No liquid-gas transition above **critical point**
- **Vapor pressure** for liquid-gas transition

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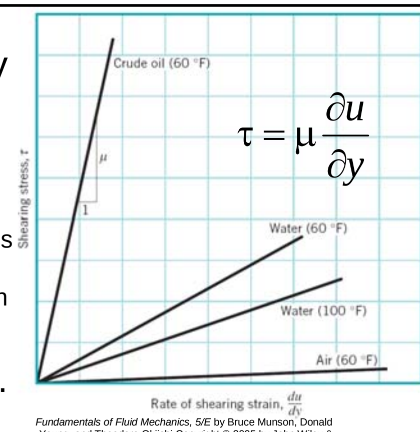
Viscosity

Newtonian Fluids have a linear variation of shearing stress with rate of shearing strain – slope is viscosity

Figure 1.4 (p. 15)



$$\tau = \mu \frac{\partial u}{\partial y}$$

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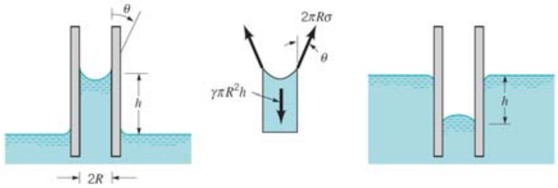
Review Surface Tension



- Vertical force balance: $\gamma\pi R^2 h = 2\pi R\sigma\cos\theta$
  - Surface tension depends on fluid, temperature; wetting angle, θ, depends on fluid and surface

$$h = \frac{2\sigma\cos\theta}{\gamma R}$$

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Pressure Relations

- Pressure is a scalar
- The force exerted by a pressure is the same in all directions
- Want to see how pressure changes in a static (nonmoving) fluid
- Look at balance of pressure force and fluid weight over a differential volume element, δxδyδz

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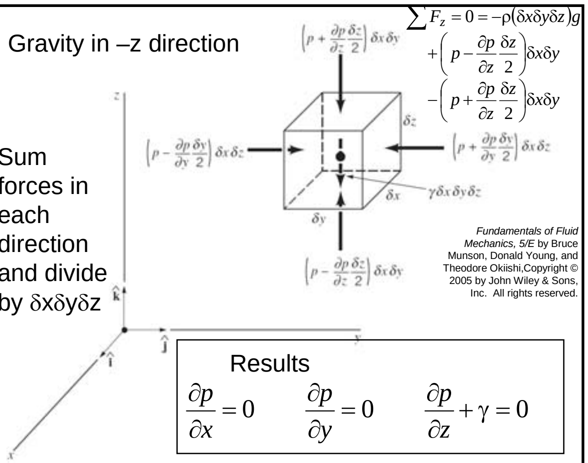
Gravity in –z direction

Sum forces in each direction and divide by δxδyδz

$$\sum F_z = 0 = -\rho(\delta x\delta y\delta z)g + \left(p - \frac{\partial p}{\partial z}\frac{\delta z}{2}\right)\delta x\delta y - \left(p + \frac{\partial p}{\partial z}\frac{\delta z}{2}\right)\delta x\delta y$$



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Results

$$\frac{\partial p}{\partial x} = 0 \qquad \frac{\partial p}{\partial y} = 0 \qquad \frac{\partial p}{\partial z} + \gamma = 0$$

Integrating the Result

- Since ∂p/∂x =∂p/∂y = 0, p = f(z) only, and we can write ∂p/∂z = –γ as dp/dz = –γ
- Multiply by dz and integrate between two points (p_1, z_1) and (p_2, z_2)

$$\int_{p_1}^{p_2} dp = p_2 - p_1 = -\int_{z_1}^{z_2} \gamma dz$$

- To integrate γdz we have to know how γ depends on z

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Incompressible Fluid

- An incompressible fluid has constant density (and specific weight)
- For an incompressible (constant density) fluid then

$$p_2 - p_1 = -\int_{z_1}^{z_2} \gamma dz = -\gamma\int_{z_1}^{z_2} dz = -\gamma(z_2 - z_1)$$

$$p_2 + \gamma z_2 = p_1 + \gamma z_1$$

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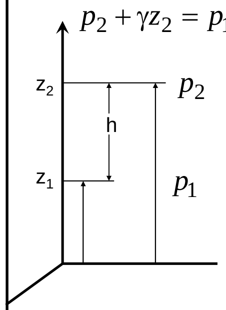
Incompressible Fluid II

$$p_2 + \gamma z_2 = p_1 + \gamma z_1$$



- Which pressure is higher?

$$p_1 = p_2 + \gamma(z_2 - z_1)$$

$$p_1 = p_2 + \gamma h > p_2$$

- Pressure increases with depth

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Problem

- If the pressure at the surface of a body of water (γ = 9789 N/m^3 at 20^{o}C) is 101 kPa, what are the pressures at depths of 10 m and 100 m?
- **Given:** p_1 = 101 kPa at z_1 = 0
- **Find:** p at z_2 = –10 m and z_3 = –100 m
- **Equation:**

$$p_2 + \gamma z_2 = p_1 + \gamma z_1 \quad \Rightarrow \quad p_2 = p_1 + \gamma(z_1 - z_2)$$

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Solution

Depth = 10 m

$$p_2 = p_1 + \gamma(z_1 - z_2) = 101\,kPa + \frac{9789\,N}{m^3}\left[0\,m - (-10\,m)\right]\frac{kPa \cdot m^2}{1000\,N} = 198.9\,kPa$$

Depth = 100 m

$$p_3 = p_1 + \gamma(z_1 - z_3) = 101\,kPa + \frac{9789\,N}{m^3}\left[0\,m - (-100\,m)\right]\frac{kPa \cdot m^2}{1000\,N} = 1080\,kPa$$

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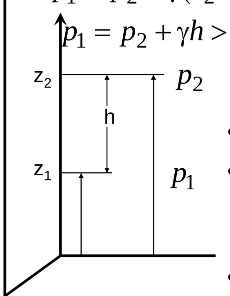
Pressure Head

$$p_1 = p_2 + \gamma(z_2 + z_1)$$

$$p_1 = p_2 + \gamma h > p_2$$



- Fluid height equivalent to a pressure difference $h = \dfrac{p_2 - p_1}{\gamma}$
- h is called pressure head
- For $p_2 - p_1$ = 14.696 psia = 101.325 kPa, h = 0.76 m = 29.92 in for Hg
- What is h for water at this Δp?

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ΔP = 1 Atm for Water at 20^{o}C

- At 20^{o}C, γ_{water} = 9789 N/m^3 = 62.32 lb$_f$/ft^3 (p 761, text)

$$h = \frac{p_2 - p_1}{\gamma} = \frac{\dfrac{14.696\,lb_f}{in^2}\dfrac{144\,in^2}{ft^2}}{\dfrac{62.32\,lb_f}{ft^3}} = 33.96\,ft$$

$$h = \frac{p_2 - p_1}{\gamma} = \frac{101.325\,kPa\dfrac{1000\,N}{kPa \cdot m^2}}{\dfrac{9789\,N}{m^3}} = 10.35\,m$$

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Free Surface

- Surface of liquid open to atmosphere is called a "free surface"
  - Pressure, p_0, is atmospheric pressure, p_0
  - Height, z_0 = 0
  - In the liquid, p + γz = p_0 + γz_0, where z < 0
  - Depth h = z_0 – z > 0
  - p = p_0 + γ(z_0 – z) = p_0 + γh
- Pressure, p, at depth, h, not influenced by size or shape of container

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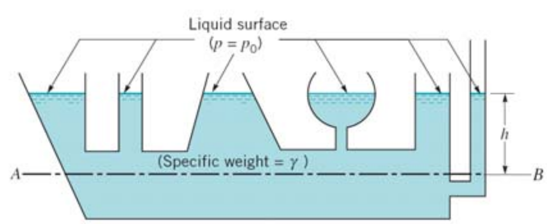
Free Surface II



- Pressure, p, at depth, h, not influenced by size or shape of container

Figure 2.4, *Fundamentals of Fluid Mechanics, 5/E* by Bruce Munson, Donald Young, and Theodore Okiishi

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Reference Pressure

- Free surface equation: $p = p_0 + \gamma(z_0 - z)$
- Apply this to two different pressures
  - $p_1 = p_0 + \gamma(z_0 - z_1)$
  - $p_2 = p_0 + \gamma(z_0 - z_2)$
- Find $p_2 - p_1$ from these equations
  - Result: $p_2 - p_1 = p_0 + \gamma(z_0 - z_2) - [p_0 + \gamma(z_0 - z_1)] = \gamma(z_1 - z_2)$ independent of p_0 or z_0
- Reference pressure cancels in taking pressure differences

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Gage Pressure

- For taking pressure differences, we can use any reference pressure
- Many pressure measurement methods measure the difference between actual and atmospheric pressure
- We can used this measured pressure difference, called gage pressure, directly in Δp calculations
- $p_{absolute} = p_{gage} + p_{atmosphere}$

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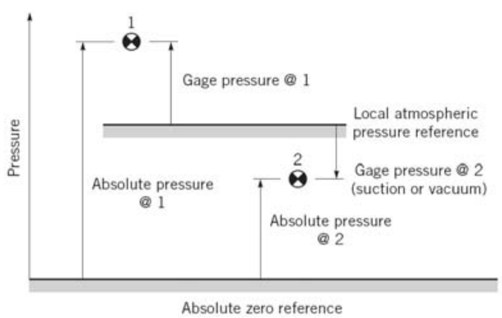
Gage and Absolute Pressure



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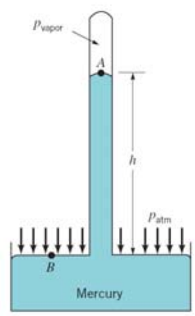
Barometric Pressure



- Mercury barometer used to measure atmospheric pressure
  - Top is evacuated and fills with mercury vapor
  - $P_{atm} = \gamma h + p_{vapor}$
  - p_{vapor} = 0.000023 psia = 0.1586 Pa at 68°F (20°C)
  - h = 760 mm = 29.92 in for standard atmosphere

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What a Barometer Measures

- It actually measures the local pressure
- A barometer in a undersea submersible, pressurized to 4 times atmospheric pressure would measure this level
- Weather barometer readings are corrected to mean sea level
- Standard atmosphere: 760 mm Hg, 760 torr, 29.921 in Hg, 101.325 kPa, 14.696 psia, 2116.2 psfa, 1013.25 mbar

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Gage/Absolute Notation

- For pressure differences a specification of gage or absolute is **not** required
- Traditional notation is psig (or psfg) and psia (or psfg) for gage and absolute pressure, respectively
- Can also use kPa(abs) or kPa(gage)
- Munson uses psi or kPa for gage pressures and psi(abs) or kPa(abs) for absolute pressures

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Variable Density

- **Problem:** integrate dp/dz = –γz when density (and hence γ) is not constant
- **Simple solution:** for gases γ is small so that p does not change much with z
  - *E. g.* air at atmospheric pressure and T = 20ºC has γ = 11.81 N/m³
  - If γ = 11.81 N/m³ were constant an elevation change of 10 m gives Δp = (11.81 N/m³)(10 m) = 118.1 N/m² = 0.1181 kPa

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Variable Density II

- **Result:** the pressure change of 0.1181 kPa is only 0.12% of p_{atm} = 101.325 kPa
- **Simple solution:** for gases with small elevation changes we can assume that the specific weight is constant!
- This is not valid for changes of several kilometers as in the atmosphere
  - Standard atmosphere defined and used for aerospace designs

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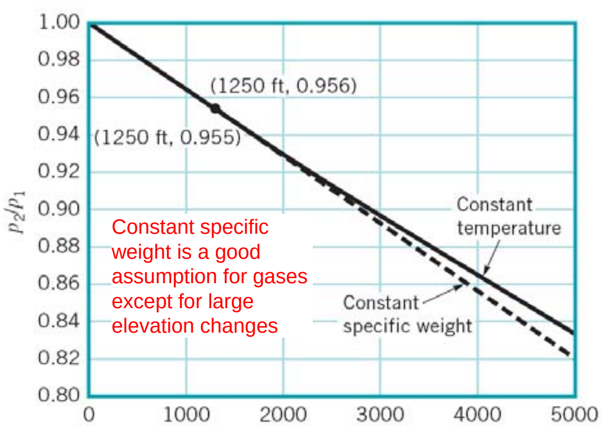

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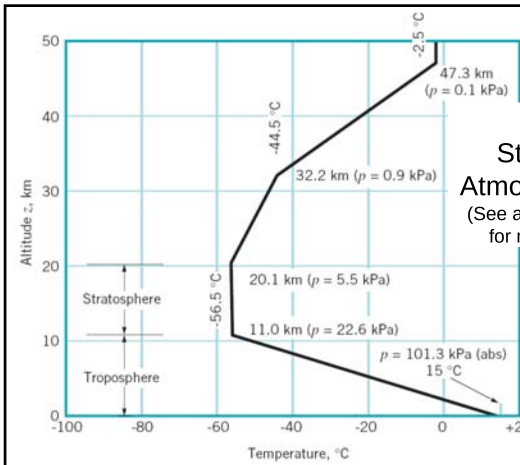

U. S. Standard Atmosphere (See appendix C for more data)

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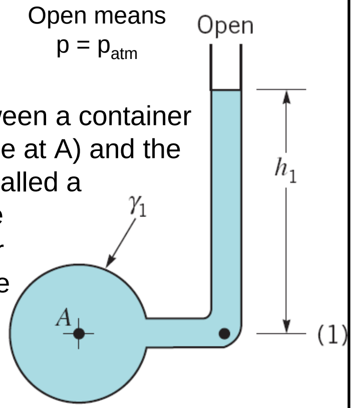
Piezometer

Open means $p = p_{atm}$

- A passage between a container (such as the pipe at A) and the atmosphere is called a piezometer tube
- For a piezometer tube the pressure in the fluid, $p_A = p_1 = p_{atm} + \gamma_1 h_1$
- p_{atm}(gage) = 0



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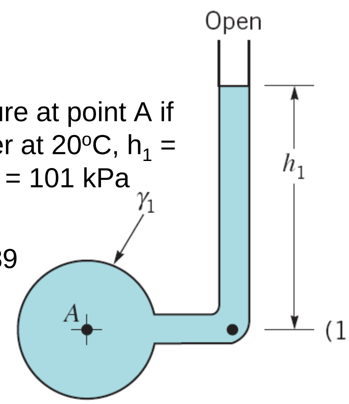
Problem

- Find the pressure at point A if the fluid is water at 20ºC, h_1 = 0.2 m, and p_{atm} = 101 kPa
- From table B.2, $\gamma_1 = \gamma_{water}$ = 9.789 kN/m³ at 20ºC
- $p_A = p_1 = p_{atm} + \gamma_1 h_1$



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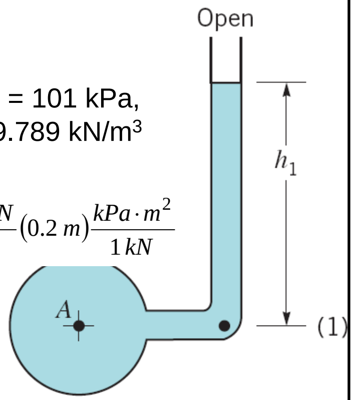
Solution

- $h_1 = 0.2$ m, $p_{atm} = 101$ kPa, and $\gamma_1 = \gamma_{water} = 9.789$ kN/m^3
- $p_A = p_{atm} + \gamma_1 h_1$

$$P_A = 101\,kPa + \frac{9.789\,kN}{m^3}(0.2\,m)\frac{kPa \cdot m^2}{1\,kN}$$

- $p_A = 103$ kPa (absolute)
- $p_A = 1.96$ kPa (gage)



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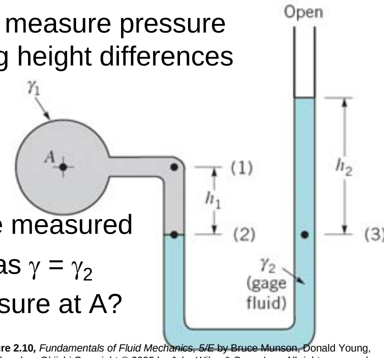
Simple U-Tube Manometer

- Manometers measure pressure by measuring height differences
- Point A is fluid ($\gamma = \gamma_1$) in a pipe
- h_1 and h_2 are measured
- Gage fluid has $\gamma = \gamma_2$
- What is pressure at A?



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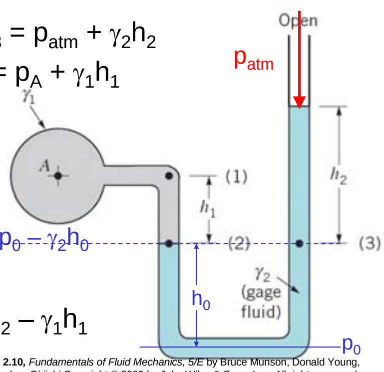
Simple U-Tube Manometer II

- Right side: $p_3 = p_{atm} + \gamma_2 h_2$
- Left side: $p_2 = p_A + \gamma_1 h_1$
- $p_3 = p_2$ gives $p_{atm} + \gamma_2 h_2 = p_A + \gamma_1 h_1$



- Conclusion: $p_A = p_{atm} + \gamma_2 h_2 - \gamma_1 h_1$

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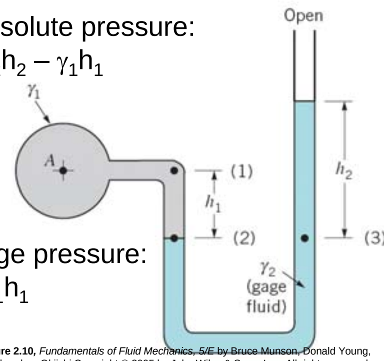
Simple U-Tube Manometer III

- Result for absolute pressure: $p_A = p_{atm} + \gamma_2 h_2 - \gamma_1 h_1$
- Result for gage pressure: $p_A = \gamma_2 h_2 - \gamma_1 h_1$



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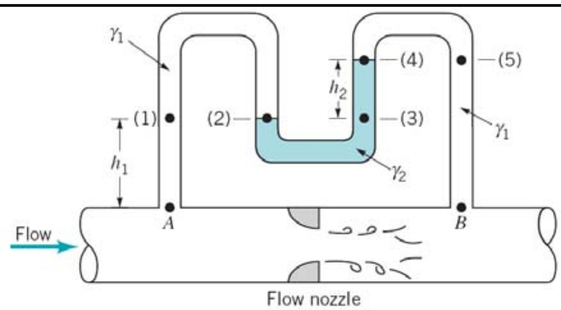

- **Given:** Known specific weights and measured heights shown above
- **Find:** $p_A - p_B$ **Equation:** $p_\alpha + \gamma z_\alpha = p_\beta + \gamma z_\beta$

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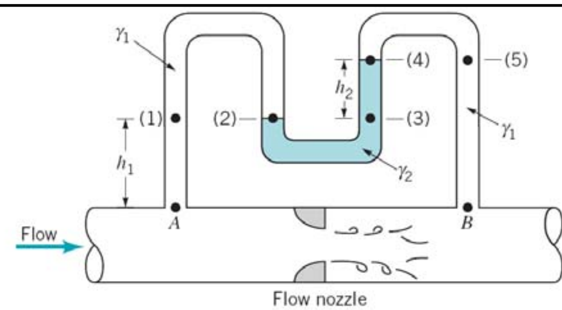

$$p_A = p_1 + \gamma_1 h_1 \quad p_1 = p_2 = p_3 \quad p_3 = p_4 + \gamma_2 h_2$$

$$p_5 = p_4 \quad p_B = p_5 + \gamma_1 (h_1 + h_2)$$

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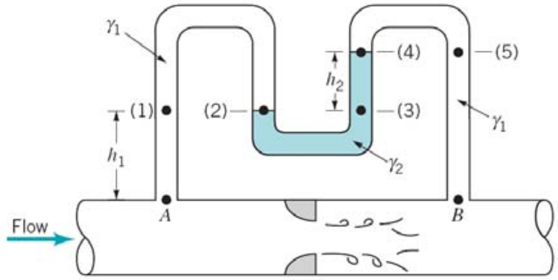

Combine first set of equations

$p_A = p_1 + \gamma_1 h_1$ $\quad p_1 = p_2 = p_3$ $\quad p_3 = p_4 + \gamma_2 h_2$

$$p_A = p_1 + \gamma_1 h_1 = p_3 + \gamma_1 h_1 = p_4 + \gamma_2 h_2 + \gamma_1 h_1$$

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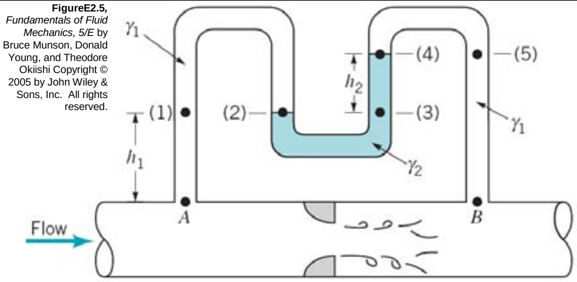

Combine result from previous page with second set of equations

$p_A = p_4 + \gamma_2 h_2 + \gamma_1 h_1$

$p_5 = p_4$ $\quad p_B = p_5 + \gamma_1 (h_1 + h_2)$

$$p_A = p_5 + \gamma_2 h_2 + \gamma_1 h_1 = p_B - \gamma_1 (h_1 - h_2) + \gamma_2 h_2 + \gamma_1 h_1$$

$$p_A - p_B = (\gamma_2 - \gamma_1) h_2$$

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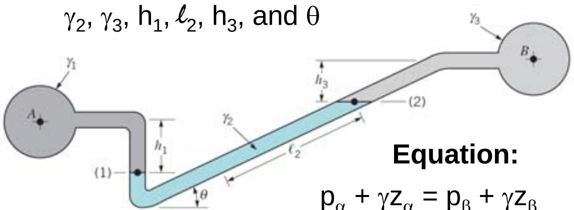
Incline Manometer Problem

- Incline used to increase accuracy for small pressure differences
- Want to find $p_B - p_A$ when we know γ_1, γ_2, γ_3, h_1, ℓ_2, h_3, and θ



Equation:

$p_\alpha + \gamma z_\alpha = p_\beta + \gamma z_\beta$

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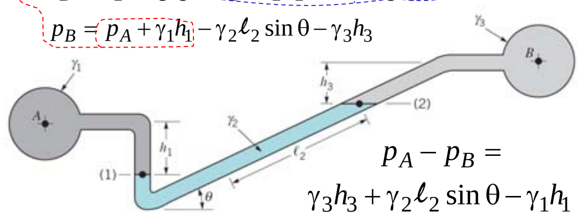
Incline Manometer Problem II

$p_1 = p_A + \gamma_1 h_1$ $\quad p_1 = p_2 + \gamma_2 (z_2 - z_1) = p_2 + \gamma_2 \ell_2 \sin\theta$

$p_2 = p_B + \gamma_3 h_3 \quad \Rightarrow \quad p_B = p_2 - \gamma_3 h_3$

$p_B = p_2 - \gamma_3 h_3 = p_1 - \gamma_2 \ell_2 \sin\theta - \gamma_3 h_3$

$p_B = p_A + \gamma_1 h_1 - \gamma_2 \ell_2 \sin\theta - \gamma_3 h_3$



$$p_A - p_B = \gamma_3 h_3 + \gamma_2 \ell_2 \sin\theta - \gamma_1 h_1$$

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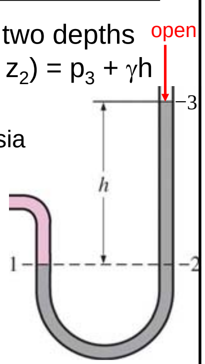
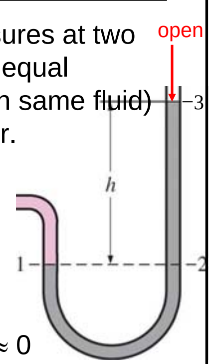
Solving Manometer Problems

- Basic equation: pressures at two depths in same fluid: $p_2 = p_3 + \gamma(z_3 - z_2) = p_3 + \gamma h$
- "Open" means $p = p_{atm}$
  - p_{atm} = 101.325 kPa = 14.696 psia
  - For gage pressure, $p_{atm} = 0$
- Same pressures at same level on two sides of a manometer
  - $p_2 = p_3$



Solving Manometer Problems II

- Write equations for (1) pressures at two depths in same fluid and (2) equal pressures at same level (with same fluid) at all branches in manometer.
- Eliminate intermediate pressures from equations to get desired ΔP
- Watch units for length, psi or psf, N or kN
  - For gases $\gamma \Delta z \approx 0$



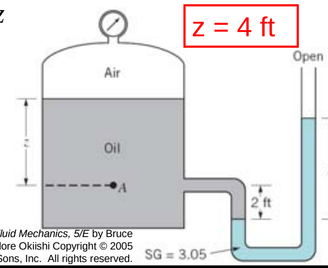
Problem

- **Given:** air at 0.50 psig, p_A = 2 psig, and γ_{oil} = 54.0 lb_f/ft^3 and other data shown on diagram. **Find:** z and h

$$p_A - p_{air} = \gamma_{oil} z$$

$$z = \frac{p_A - p_{air}}{\gamma_{oil}} = \frac{\frac{2\ lb_f}{in^2} - \frac{0.5\ lb_f}{in^2}}{\frac{54.0\ lb_f}{ft^3}\frac{ft^2}{144\ in^2}}$$

z = 4 ft



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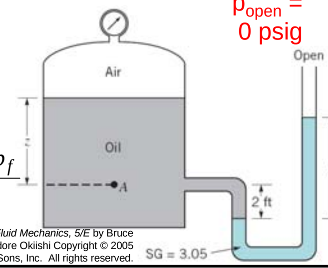
Problem Continued

- **Given:** air at 0.50 psig, p_A = 2 psig, and γ_{oil} = 54.0 lb_f/ft^3 and other data shown on diagram. **Find:** h

$$p_A + \gamma_{oil}(2\ ft) = p_{open} + \gamma_{mano} h$$

p_{open} = 0 psig

$$\gamma_{mano} = SG_{mano}\gamma_{H_2O} = 3.05\left(\frac{62.4\ lb_f}{ft^3}\right) = \frac{190.3\ lb_f}{ft^3}$$



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Problem Concluded

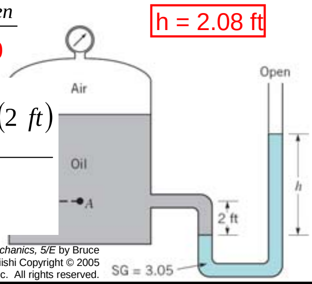
- p_{open} = 0 psig, γ_{mano} = 190.3 lb_f/ft^3, h = ?

$$p_A + \gamma_{oil}(2\ ft) = p_{open} + \gamma_{mano} h$$

$$h = \frac{p_A + \gamma_{oil}(2\ ft) - p_{open}}{\gamma_{mano}} \quad (p_{open} \to 0)$$

$$h = \frac{\frac{2\ lb_f}{in^2}\frac{144\ in^2}{ft^2} + \frac{54.0\ lb_f}{ft^3}(2\ ft)}{\frac{190.3\ lb_f}{ft^3}}$$

h = 2.08 ft



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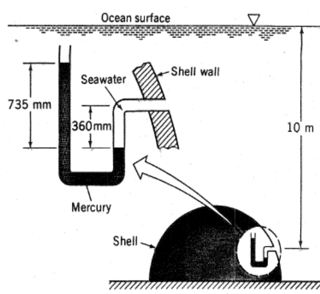
Problem 2.38



A hemispherical shell on the ocean floor has an internal barometric pressure of 765 mm Hg. A mercury manometer measures the differential pressure between the sea outside and the shell interior as shown in the diagram.

What is the pressure at the ocean surface?

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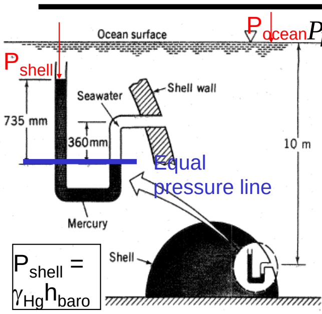
Problem 2.38 Part 2



P_{shell} = $\gamma_{Hg}h_{baro}$

h_{baro} = 765 mm Hg

$$P_{left} = P_{shell} + \gamma_{Hg}(0.735\ m) = P_{right} = P_{ocean} + \gamma_{sea\ water}(10\ m + 0.36\ m)$$

Use specific weight data from Table 1-6 (ignore difference in temperature)

γ_{Hg} = 133 kN/m³

$\gamma_{sea\ water}$ = 10.1 kN/m³

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Problem 2.38 Part 3

- Rearrange equations from previous slide and substitute given data

$$P_{left} = P_{shell} + \gamma_{Hg}(0.735\ m) = P_{right} = P_{ocean} + \gamma_{sea\ water}(10.36\ m)$$

$$P_{ocean} = P_{shell} + \gamma_{Hg}(0.735\ m) - \gamma_{sea\ water}(10.36\ m)$$

$$P_{shell} = \gamma_{Hg}h_{baro} = \left(\frac{133\ kN}{m^3}\right)(0.765\ m) = \frac{101.745\ kN}{m^2}$$

$$P_{ocean} = \frac{101.745\ kN}{m^3} + \frac{133\ kN}{m^3}(0.735\ m) - \frac{10.1\ kN}{m^3}(10.36\ m)$$

$$P_{ocean} = \frac{94.9\ kN}{m^2} = 94.9\ kPa$$

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