Laboratory V – Program Control Using if Statements

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Outline

- Review first quiz
- Summarize lecture material on if statements and Boolean variables
- Exercise five goals
- Outline tasks for exercise five
- Provide details for some tasks
- Note – exercises five to eight have two times the credit of exercises one to four

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Exercise Five Goals

- As a result of this exercise you should be able to accomplish the following:
  - recognize two common errors in if statements
  - use relational operators, Boolean operators and Boolean variables
  - write a program from pseudocode using if statements
  - understand effect of round-off error and use of algorithms to avoid such error

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First Quiz Results

- 10 students
- 60 maximum possible
- 44.9 mean
- 45.5 median
- 9.13 standard deviation
- Grade distribution

```
25 39 41 41 45 46 50 51 55 56
```

- Solutions available on line

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Questions One and Two

- Most errors on integer operations in first question
  - Integer division truncates regardless of variable on left hand side
  - Statement int(a) truncates
- Second question generally okay
  - Remember to use * for multiplication
  - Check parentheses for correctness
  - Remember to use pow(number, power) for exponentiation

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Question Three

```
#include <iostream>
#include <cmath>
using namespace std;
int main()
{
  cout << "This program computes "
       << "trajectory parameters\n\n"
       << "Enter the initial velocity in "
       << "meters per second: ";
   double v0;
   cin >> v0;
   const double g = 9.807;
```

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Question Three, Part Two

```
    double hMax = v0 * v0 / g;
    double tMax = v0 / g;
    double tFinal = 2 * v0 / g;
    cout << "\n\nThe maximum height is "
         << hMax << " meters."
         << "\nThe maximum height occurs"
         << " at " << tMax << " seconds."
         << "\nThe final time is " << tFinal
         << " seconds.";
    return EXIT_SUCCESS;
}
```


Tasks for Exercise Five

- One − copy and paste code with errors in if statements; run without changes, correct errors and run corrected code
- Two − copy, paste and run code using Boolean variables for data validation
- Three − write a program with if statements to find roots of quadratic equations handling exceptional conditions
- Four − modify task three code to reduce round-off error

Task 1: Errors in if Statements

- Semicolon after condition parentheses
  - `if ( <condition> );  // error`
- Assignment operator (=) instead of relational equality operator (==)
  - `if ( x = 3 ) //usually an error`
- When you use (a = b) as a condition
  - the value of a is set to the value of b
  - the resulting value of a is converted to bool
    - zero is false; nonzero is true

Task 2: Boolean Variables

- Declared as type bool
- Used for complicated conditional tests
- Have values false (0) or true (not zero)
- Some possible statements
  - int x, xMax = 10000, xMin = 100;
  - cout << "Enter x: "; cin >> x;
  - bool badData = x < xMin || x > xMax;
  - bool goodData = !badData;

The `boolalpha` Manipulator

- Output of Boolean variables normally prints 1 for true and 0 for false
- `boolalpha` manipulator prints the text true or false
  - bool good = true; bad = !good;
  - cout << good << " " << bad; // prints 1 0
  - cout << boolalpha << good << " " << bad; // prints true false
- Use `noboolalpha` to return to default

Tasks 3 and 4: Quadratic Roots

$$x_+ = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad x_- = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

- Possible problems with these solutions to $ax^2 + bx + c = 0$ in a computer
  - a = b = 0 and a = 0, b ≠ 0
  - complex roots
  - two roots the same
  - roundoff errors

When is b^2 equal to 4ac?

- Numbers are not represented exactly in the computer (roundoff error)
- Smallest double such that 1 + x != 1 is 2.2204460492503131e–016
- Test for $b^2 - 4ac = 0$ should be written as follows

```
const double eps = 1e-15;
if ( fabs( b * b – 4 * a * c )
  <= eps * (b * b + fabs(4 * a *
  c) );
```


Initial Pseudocode for Quadratics

- If a = b = 0
  - Print out that there is no solution
- Otherwise, if a = 0 and b ≠ 0,
  - Print message that this is a linear equation with only one solution; print solution of –c/b
- Otherwise if $b^2 - 4ac \approx 0$ (see previous chart for this test)
  - Print message that the two roots are the same and print the common root, –b/(2a)

Initial Quadratic Pseudocode II

- Otherwise, if $b^2 - 4ac < 0$
  - Print message that roots are complex and print two complex roots
- Otherwise
  - Compute roots from conventional formulas shown below
  - Print message that there are two distinct roots and print values of roots

$$x_+ = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad x_- = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Printing Complex Roots

- Complex roots for $b^2 < 4ac$ ($i^2 = -1$)

$$x_1 = \underbrace{\frac{-b}{2a}}_{\text{Real part}} + i\underbrace{\frac{\sqrt{|b^2 - 4ac|}}{2a}}_{\text{Imaginary part}} \qquad x_2 = \frac{-b}{2a} - i\frac{\sqrt{|b^2 - 4ac|}}{2a}$$

- Take absolute value (fabs in C++) of $b^2 - 4ac$ before taking square root
- Print roots one of two ways
  - label real part and imaginary part or print roots as x1 = 3 + i4 and x2 = 3 – i4
  - Use cout << " + i" or " – i" in latter option

Task Three Cases

- Seven different cases shown below and in instructions
- Run all cases and print case VII results with 14 significant figures for comparison with task four results

case	I	II	III	IV	V	VI	VII
a	0	0	0	2	1	1	10^{-6}
b	0	0	1.5	0	2	4	1
c	0	1	2.25	1	1	1	10^{-6}

More on Roundoff Error

- Arises because numbers are not represented exactly in a computer
- Main problem is the "subtraction" of two nearly equal numbers

  123456789 – 123456788 = 1

  123456789 + (–123456788) = 1
  - Reduce significant figures from nine to one
  - Algorithm for quadratic equation solutions can avoid this

Example, $\sqrt{b^2 - 4ac} = 123456788$

$-b \pm \sqrt{b^2 - 4ac}$

- If b = 123456789 use of + part of ± sign will give roundoff but – sign is okay
- If b = -123456789 use of – part of ± sign will give roundoff but + sign is okay
- We know that the product of the two roots of a quadratic equals c/a
- Choose formula for root x_1 with + or – to avoid roundoff then compute second from $x_2 = c/(ax_1)$

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How do we know $x_1x_2 = c/a$?

$$x_1x_2 = \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

- Use $(x + y)(x – y) = x^2 – y^2$ and $\left(\sqrt{z}\right)^2 = z$

$$x_1x_2 = \frac{(-b)^2 - \left(\sqrt{b^2-4ac}\right)^2}{4a^2} =$$

$$\frac{b^2 - \left(b^2-4ac\right)}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}$$

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Final Pseudocode for Quadratics

- If a = b = 0 (changes in red)
  - Print out that there is no solution
- Otherwise, if a = 0 and b ≠ 0,
  - Print message that there is only one solution and print solution of –c/b
- Otherwise if $b^2 – 4ac \approx 0$
  - Print message that the two roots are the same and print the common root, –b/(2a)

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Final Quadratic Pseudocode II

- Otherwise, if $b^2 – 4ac < 0$
  - Print message that roots are complex and print two complex roots
- Otherwise
  - If b is positive
    - Compute first root $x_1 = [–b – (b^2 – 4ac)^{1/2}]/(2a)$
  - otherwise
    - Compute first root, $x_1 = [–b + (b^2 – 4ac)^{1/2}]/(2a)$
  - End of "If b is positive" choices
  - Compute second root $x_2 = c/(ax_1)$
  - Print message and two distinct roots

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Compare Results

- Run case VII input with both task three and task four code
- Get output with 14 significant figures
- Compare results from the two codes with the "exact" answer from series expansion in notes.

$$x_1 = \frac{b}{a}\left(-1 + \frac{ac}{b^2} + \frac{a^2c^2}{b^4} + \frac{2a^3c^3}{b^6} + \cdots\right)$$

$$x_2 = \frac{c}{b}\left(1 + \frac{ac}{b^2} + \frac{2a^2c^2}{b^4} + \frac{5a^3c^3}{b^6} + \cdots\right)$$

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Compare Results II

- Task three has seven data sets
- Use data set VII only for task four
- For this data set, use 14 significant figures of accuracy to compare
  - Results from task three
  - Results from task four
  - Results from series expansion
- You should find that the task four code is more accurate for one of the roots

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Good Style: Indent Structures

```
if ( hours > 40 )
{
    pay = rate * ( 40 +
           1.5 * ( hours - 40 ) );
}
else
{
    pay = rate * hours;
}
```

Bad Structure

```
if(hours>40){pay=rate*(40+1.5*(hours
-40));}else{pay=rate*hours;}
```

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25