Steffensen's Method

```
> restart;
```

```
> g:=x->(10/(4+x))^(1/2);
```

$$g := x \mapsto \sqrt{10}\,\sqrt{\frac{1}{4+x}} \tag{1}$$

```
> p0:=1.5;
```

$$p0 := 1.5 \tag{2}$$

```
> for n from 1 to 20 do
  p[n]:=evalf(g(p0));
  err:=abs(p[n]-p0);
  if err>=10^(-10) then
  p0:=p[n];
  else
  break
  end if
  end do;
```

$$p_1 := 1.348399725$$

$$err := 0.151600275$$

$$p_2 := 1.367376372$$

$$err := 0.018976647$$

$$p_3 := 1.364957015$$

$$err := 0.002419357$$

$$p_4 := 1.365264748$$

$$err := 0.000307733$$

$$p_5 := 1.365225594$$

$$err := 0.000039154$$

$$p_6 := 1.365230576$$

$$err := 4.982\ 10^{-6}$$

$$p_7 := 1.365229942$$

$$err := 6.34\ 10^{-7}$$

$$p_8 := 1.365230022$$

$$err := 8.0\ 10^{-8}$$

$$p_9 := 1.365230012$$

$$err := 1.0\ 10^{-8}$$

$$p_{10} := 1.365230014$$

$$err := 2.\ 10^{-9}$$

$$p_{11} := 1.365230013$$

$$err := 1.\ 10^{-9}$$

$$p_{12} := 1.365230014$$
$$err := 1.\,10^{-9}$$
$$p_{13} := 1.365230013$$
$$err := 1.\,10^{-9}$$
$$p_{14} := 1.365230014$$
$$err := 1.\,10^{-9}$$
$$p_{15} := 1.365230013$$
$$err := 1.\,10^{-9}$$
$$p_{16} := 1.365230014$$
$$err := 1.\,10^{-9}$$
$$p_{17} := 1.365230013$$
$$err := 1.\,10^{-9}$$
$$p_{18} := 1.365230014$$
$$err := 1.\,10^{-9}$$
$$p_{19} := 1.365230013$$
$$err := 1.\,10^{-9}$$
$$p_{20} := 1.365230014$$
$$err := 1.\,10^{-9} \quad \textbf{(3)}$$

```
> P0:=1.5;
```

$$P0 := 1.5 \quad \textbf{(4)}$$

```
> for n from 1 to 20 do
  P1:=evalf(g(P0)); P2:=evalf(g(P1));
  p[n]:=P0-(P1-P0)^2/(P2-2*P1+P0);
  err:=abs(p[n]-P0);
  if err>=10^(-10) then
  P0:=p[n];
  else
  break
  end if
  end do;
```

$$P1 := 1.348399725$$
$$P2 := 1.367376372$$
$$p_{1} := 1.365265224$$
$$err := 0.134734776$$
$$P1 := 1.365225534$$

$$P2 := 1.365230583$$
$$p_2 := 1.365230013$$
$$err := 0.000035211$$
$$P1 := 1.365230014$$
$$P2 := 1.365230013$$
$$p_3 := 1.365230014$$
$$err := 1.\ 10^{-9}$$
$$P1 := 1.365230013$$
$$P2 := 1.365230014$$
$$p_4 := 1.365230014$$
$$err := 0. \tag{5}$$

```
> solve(g(x)=x,x);
```

$$\sqrt{10}\left(\frac{\left(710\sqrt{10}+150\sqrt{42}\right)^{1/3}}{30}+\frac{16}{3\left(710\sqrt{10}+150\sqrt{42}\right)^{1/3}}-\frac{2\sqrt{10}}{15}\right) \tag{6}$$

```
> evalf(%);
```

$$1.365230014 \tag{7}$$

```
> g1:=x->x-(g(x)-x)^2/(g(g(x))-2*g(x)+x);
```

$$g1 := x \mapsto x-\frac{\left(g(x)-x\right)^2}{g(g(x))-2\,g(x)+x} \tag{8}$$

```
> p0:=1.5;
```

$$p0 := 1.5 \tag{9}$$

```
> for n from 1 to 20 do
  p[n]:=evalf(g1(p0));
  err:=abs(p[n]-p0);
  if err>=10^(-10) then
  p0:=p[n];
  else
  break
  end if
  end do;
```

$$p_1 := 1.365265224$$
$$err := 0.134734776$$
$$p_2 := 1.365230014$$
$$err := 0.000035210$$
$$p_3 := 1.365230014$$
$$err := 0. \tag{10}$$

```
> restart;
```

```
> g:=x->cos(x);
```

$$g := x \mapsto \cos(x) \tag{11}$$

```
> g1:=x->x-(g(x)-x)^2/(g(g(x))-2*g(x)+x);
```

$$g1 := x \mapsto x - \frac{(g(x) - x)^2}{g(g(x)) - 2\,g(x) + x} \tag{12}$$

```
> p0:=1.0;
```

$$p0 := 1.0 \tag{13}$$

```
> for n from 1 to 20 do
  p[n]:=evalf(g1(p0));
  err:=abs(p[n]-p0);
  if err>=10^(-10) then
  p0:=p[n];
  else
  break
  end if
  end do;
```

$$p_1 := 0.7280103613$$

$$err := 0.2719896387$$

$$p_2 := 0.7390669670$$

$$err := 0.0110566057$$

$$p_3 := 0.7390851332$$

$$err := 0.0000181662$$

$$p_4 := 0.7390851332$$

$$err := 0. \tag{14}$$

```
> fsolve(cos(x)=x,x=1.0);
```

$$0.7390851332 \tag{15}$$