

## A non-converging example:

```
> restart; with(plots):
```

```
> f:=x->1-x^2;
```

$$f:=x \rightarrow 1 - x^2$$

(1)

```
> sol:=solve({1-x^2=x},{x});
```

$$\text{sol}:=\left\{x = -\frac{1}{2}\sqrt{5} - \frac{1}{2}\right\}, \left\{x = \frac{1}{2}\sqrt{5} - \frac{1}{2}\right\}$$

(2)

```
> evalf({sol[1],sol[2]});
```

$$\{x = -1.618033988\}, \{x = 0.6180339880\}$$

(3)

```
> # A[0]:=evalf(-1/2+sqrt(5)/2);
```

```
> A[0]:=0.6180339880;
```

$$A_0 := 0.6180339880$$

(4)

```
> for n from 0 to 100 do
```

```
  A[n+1]:=f(A[n])
```

```
end do;
```

$$A_1 := 0.6180339897$$

$$A_2 := 0.6180339876$$

$$A_3 := 0.6180339902$$

$$A_4 := 0.6180339870$$

$$A_5 := 0.6180339909$$

$$A_6 := 0.6180339861$$

$$A_7 := 0.6180339920$$

$$A_8 := 0.6180339847$$

$$A_9 := 0.6180339938$$

$$A_{10} := 0.6180339825$$

$$A_{11} := 0.6180339965$$

$$A_{12} := 0.6180339792$$

$$A_{13} := 0.6180340006$$

$$A_{14} := 0.6180339741$$

$$A_{15} := 0.6180340069$$

$$A_{16} := 0.6180339663$$

$$A_{17} := 0.6180340165$$

$$A_{18} := 0.6180339544$$

$$A_{19} := 0.6180340312$$

$$A_{20} := 0.6180339363$$

$$A_{21} := 0.6180340536$$

$$A_{22} := 0.6180339086$$

$$A_{23} := 0.6180340878$$

$$A_{24} := 0.6180338663$$

$A_{25} := 0.6180341401$   
 $A_{26} := 0.6180338017$   
 $A_{27} := 0.6180342200$   
 $A_{28} := 0.6180337029$   
 $A_{29} := 0.6180343421$   
 $A_{30} := 0.6180335520$   
 $A_{31} := 0.6180345286$   
 $A_{32} := 0.6180333215$   
 $A_{33} := 0.6180348135$   
 $A_{34} := 0.6180329693$   
 $A_{35} := 0.6180352489$   
 $A_{36} := 0.6180324311$   
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 $A_{39} := 0.6180369304$   
 $A_{40} := 0.6180303527$   
 $A_{41} := 0.6180384831$   
 $A_{42} := 0.6180284334$   
 $A_{43} := 0.6180408555$   
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 $A_{47} := 0.6180500182$   
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 $A_{49} := 0.6180584795$   
 $A_{50} := 0.6180037159$   
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 $A_{53} := 0.6180911585$   
 $A_{54} := 0.6179633198$   
 $A_{55} := 0.6181213354$   
 $A_{56} := 0.6179260147$   
 $A_{57} := 0.6181674404$   
 $A_{58} := 0.6178690156$   
 $A_{59} := 0.6182378796$   
 $A_{60} := 0.6177819242$   
 $A_{61} := 0.6183454941$   
 $A_{62} := 0.6176488499$   
 $A_{63} := 0.6185098982$   
 $A_{64} := 0.6174455058$

$A_{65} := 0.6187610474$   
 $A_{66} := 0.6171347662$   
 $A_{67} := 0.6191446803$   
 $A_{68} := 0.6166598649$   
 $A_{69} := 0.6197306110$   
 $A_{70} := 0.6159339698$   
 $A_{71} := 0.6206253448$   
 $A_{72} := 0.6148241814$   
 $A_{73} := 0.6219912260$   
 $A_{74} := 0.6131269148$   
 $A_{75} := 0.6240753863$   
 $A_{76} := 0.6105299122$   
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 $A_{79} := 0.6320929850$   
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 $A_{87} := 0.6924267109$   
 $A_{88} := 0.5205452500$   
 $A_{89} := 0.7290326427$   
 $A_{90} := 0.4685114059$   
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 $A_{97} := 0.9766256668$   
 $A_{98} := 0.0462023069$   
 $A_{99} := 0.9978653468$   
 $A_{100} := 0.0042647497$   
 $A_{101} := 0.9999818119$

(5)

> L:= [seq(seq([A[n],A[n+i]],i=0..1),n=1..99)]:

L1:= [[A[0],A[1]],op(L)];

L1:= [[0.6180339880, 0.6180339897], [0.6180339897, 0.6180339897],

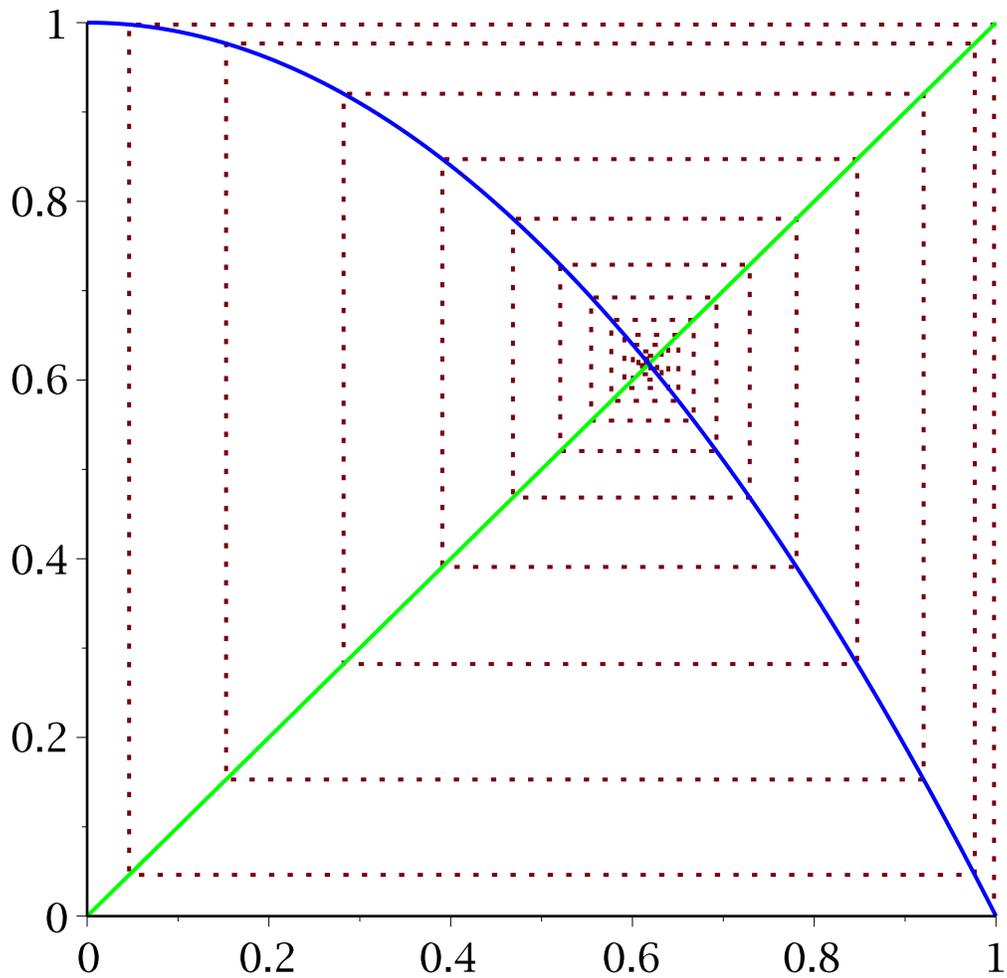
(6)

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```

```
> G1:=plot(L1,linestyle=[DOT]):
> G2:=plot({x,1-x^2},x=0..1,color=[green,blue]):
> display([G1,G2]);
```



And a converging example:

```
> g:=x->cos(x); A[0]:=1.5;
> #g:=x->x+(1/6)*x^2-(1/2): A[0]:=1.0:
      g:=x->cos(x)
      A0:=1.5
```

```
> for n from 0 to 24 do
  A[n+1]:=g(A[n])
end do;
```

```
A1:= 0.07073720167
A2:= 0.9974991672
```

(7)

$A_3 := 0.5424049923$   
 $A_4 := 0.8564697090$   
 $A_5 := 0.6551088017$   
 $A_6 := 0.7929816458$   
 $A_7 := 0.7017241683$   
 $A_8 := 0.7637303113$   
 $A_9 := 0.7222610821$   
 $A_{10} := 0.7503128857$   
 $A_{11} := 0.7314755580$   
 $A_{12} := 0.7441895867$   
 $A_{13} := 0.7356370983$   
 $A_{14} := 0.7414033729$   
 $A_{15} := 0.7375215545$   
 $A_{16} := 0.7401374748$   
 $A_{17} := 0.7383758542$   
 $A_{18} := 0.7395627261$   
 $A_{19} := 0.7387633366$   
 $A_{20} := 0.7393018610$   
 $A_{21} := 0.7389391254$   
 $A_{22} := 0.7391834780$   
 $A_{23} := 0.7390188834$   
 $A_{24} := 0.7391297583$   
 $A_{25} := 0.7390550725$

(8)

> L2:= [seq(seq([A[n],A[n+i]],i=0..1),n=1..23)]:

L3:= [[A[0],A[1]],op(L2)];

L3:= [[1.5, 0.07073720167], [0.07073720167, 0.07073720167],  
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(9)

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```

```
> H1:=plot(L3,linestyle=[DOT]):
```

```
> H2:=plot({x,g(x)},x=0..1.6,color=[green,blue]):
```

```
> # H2:=plot({x,g(x)},x=-2..2,color=[green,blue]):
```

```
> display([H1,H2]);
```

