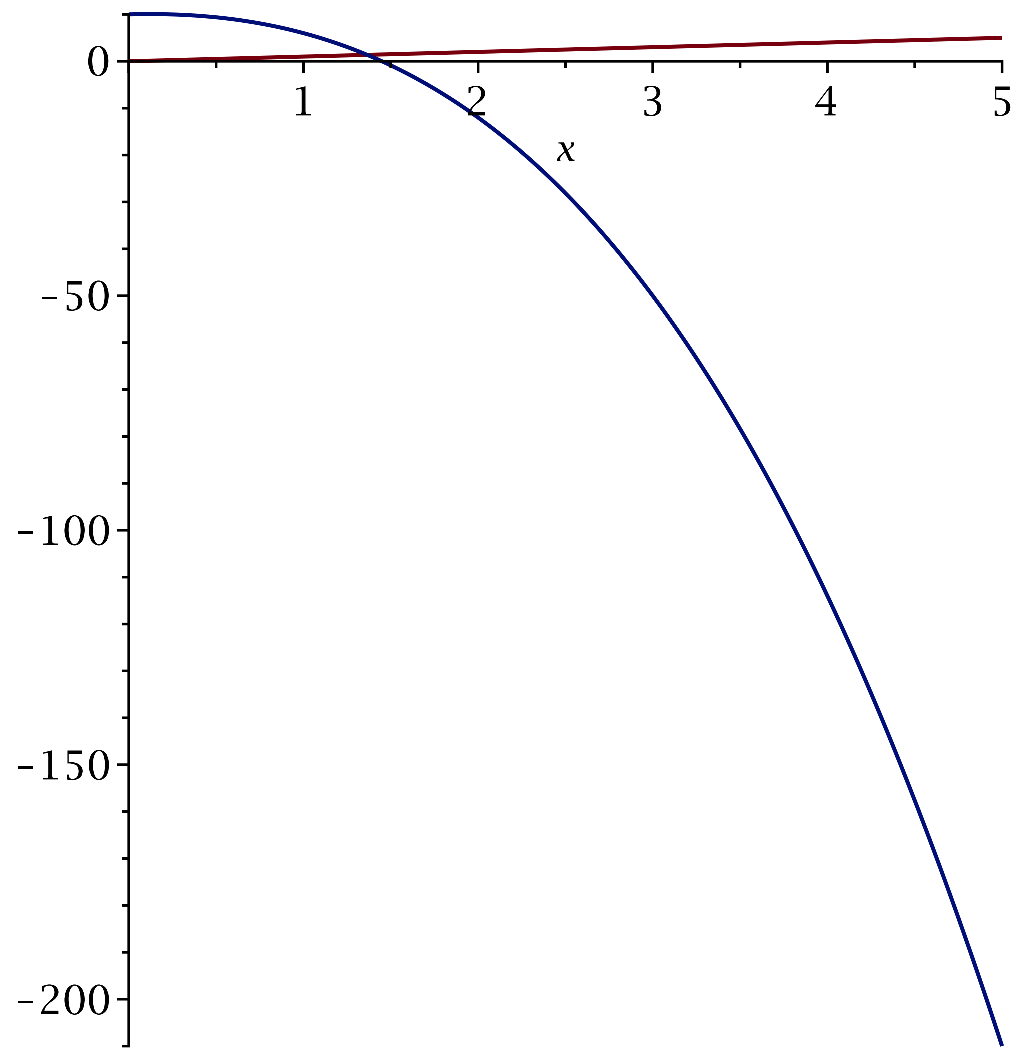
Fixed-point iteration approximations

Illustration (a)

```
> restart;
g1:=x->x-x^3-4*x^2+10;
```

$$g1 := x \mapsto -x^3 - 4\,x^2 + x + 10 \tag{1}$$

```
> plot({g1(x),x},x=0..5);
```




```
> p0:=1.5;
```

$$p0 := 1.5 \tag{2}$$

```
> for n from 1 to 20 do
  p[n]:=g1(p0);
  err:=abs(p[n]-p0);
  if err>=10^(-4) then
  p0:=p[n];
  else
  break
```

```
end if
end do;
```

$$p_1 := -0.875$$
$$err := 2.375$$
$$p_2 := 6.732421875$$
$$err := 7.607421875$$
$$p_3 := -469.7200120$$
$$err := 476.4524339$$
$$p_4 := 1.027545552\ 10^{8}$$
$$err := 1.027550249\ 10^{8}$$
$$p_5 := -1.084933871\ 10^{24}$$
$$err := 1.084933871\ 10^{24}$$
$$p_6 := 1.277055593\ 10^{72}$$
$$err := 1.277055593\ 10^{72}$$
$$p_7 := -2.082712916\ 10^{216}$$
$$err := 2.082712916\ 10^{216}$$
$$p_8 := 9.034169425\ 10^{648}$$
$$err := 9.034169425\ 10^{648}$$
$$p_9 := -7.373347340\ 10^{1946}$$
$$err := 7.373347340\ 10^{1946}$$
$$p_{10} := 4.008612522\ 10^{5840}$$
$$err := 4.008612522\ 10^{5840}$$
$$p_{11} := -6.441429180\ 10^{17521}$$
$$err := 6.441429180\ 10^{17521}$$
$$p_{12} := 2.672678432\ 10^{52565}$$
$$err := 2.672678432\ 10^{52565}$$
$$p_{13} := -1.909150330\ 10^{157696}$$
$$err := 1.909150330\ 10^{157696}$$
$$p_{14} := 6.958576093\ 10^{473088}$$
$$err := 6.958576093\ 10^{473088}$$
$$p_{15} := -3.369466493\ 10^{1419266}$$
$$err := 3.369466493\ 10^{1419266}$$
$$p_{16} := 3.825457892\ 10^{4257799}$$
$$err := 3.825457892\ 10^{4257799}$$
$$p_{17} := -5.598224077\ 10^{12773398}$$
$$err := 5.598224077\ 10^{12773398}$$
$$p_{18} := 1.754489741\ 10^{38320196}$$
$$err := 1.754489741\ 10^{38320196}$$
$$p_{19} := -5.400730414\ 10^{114960588}$$
$$err := 5.400730414\ 10^{114960588}$$

$$p_{20} := 1.575279053\ 10^{344881766}$$
$$err := 1.575279053\ 10^{344881766} \qquad (3)$$

Illustration (b)

```
> restart;
> g2:=x->(10/x-4*x)^(1/2);
```

$$g2 := x \mapsto \sqrt{\frac{10}{x} - 4\,x} \qquad (4)$$

```
> p0:=1.5;
```

$$p0 := 1.5 \qquad (5)$$

```
> for n from 1 to 20 do
  p[n]:=g2(p0);
  err:=abs(p[n]-p0);
  if err>=10^(-4) then
  p0:=p[n];
  else
  break
  end if
  end do;
```

$$p_1 := 0.8164965811$$
$$err := 0.6835034189$$
$$p_2 := 2.996908805$$
$$err := 2.180412224$$
$$p_3 := 2.941235061\,\mathrm{I}$$
$$err := 4.199086337$$
$$p_4 := 2.753622388 - 2.753622388\,\mathrm{I}$$
$$err := 6.325649186$$
$$p_5 := 1.814991519 + 3.534528789\,\mathrm{I}$$
$$err := 6.357819841$$
$$p_6 := 2.384265848 - 3.434388064\,\mathrm{I}$$
$$err := 6.992129530$$
$$p_7 := 2.182771901 + 3.596879228\,\mathrm{I}$$
$$err := 7.034153790$$
$$p_8 := 2.296997586 - 3.574104462\,\mathrm{I}$$
$$err := 7.171893375$$
$$p_9 := 2.256510286 + 3.606561220\,\mathrm{I}$$
$$err := 7.180779822$$
$$p_{10} := 2.279179049 - 3.601936572\,\mathrm{I}$$
$$err := 7.208533435$$
$$p_{11} := 2.271142587 + 3.608371470\,\mathrm{I}$$
$$err := 7.210312521$$
$$p_{12} := 2.275631312 - 3.607451621\,\mathrm{I}$$
$$err := 7.215824487$$

$$p_{13} := 2.274039927 + 3.608725567\,\mathrm{I}$$
$$err := 7.216177363$$
$$p_{14} := 2.274928362 - 3.608543344\,\mathrm{I}$$
$$err := 7.217268966$$
$$p_{15} := 2.274613385 + 3.608795481\,\mathrm{I}$$
$$err := 7.217338832$$
$$p_{16} := 2.274789212 - 3.608759412\,\mathrm{I}$$
$$err := 7.217554895$$
$$p_{17} := 2.274726876 + 3.608809311\,\mathrm{I}$$
$$err := 7.217568723$$
$$p_{18} := 2.274761673 - 3.608802172\,\mathrm{I}$$
$$err := 7.217611483$$
$$p_{19} := 2.274749336 + 3.608812048\,\mathrm{I}$$
$$err := 7.217614220$$
$$p_{20} := 2.274756223 - 3.608810635\,\mathrm{I}$$
$$err := 7.217622683 \qquad \textbf{(6)}$$

Illustration (c)

```
> restart;
> g3:=x->(1/2)*(10-x^3)^(1/2);
```

$$g3 := x \mapsto \frac{\sqrt{-x^3+10}}{2} \qquad \textbf{(7)}$$

```
> p0:=1.5;
```

$$p0 := 1.5 \qquad \textbf{(8)}$$

```
> for n from 1 to 20 do
  p[n]:=g3(p0);
  err:=abs(p[n]-p0);
  if err>=10^(-4) then
  p0:=p[n];
  else
  break
  end if
  end do;
```

$$p_1 := 1.286953768$$
$$err := 0.213046232$$
$$p_2 := 1.402540804$$
$$err := 0.115587036$$
$$p_3 := 1.345458374$$
$$err := 0.057082430$$
$$p_4 := 1.375170253$$
$$err := 0.029711879$$
$$p_5 := 1.360094192$$
$$err := 0.015076061$$

$$p_6 := 1.367846968$$
$$err := 0.007752776$$
$$p_7 := 1.363887004$$
$$err := 0.003959964$$
$$p_8 := 1.365916734$$
$$err := 0.002029730$$
$$p_9 := 1.364878217$$
$$err := 0.001038517$$
$$p_{10} := 1.365410062$$
$$err := 0.000531845$$
$$p_{11} := 1.365137820$$
$$err := 0.000272242$$
$$p_{12} := 1.365277209$$
$$err := 0.000139389$$
$$p_{13} := 1.365205850$$
$$err := 0.000071359 \quad \textbf{(9)}$$

Illustration (d)

```
> restart;
> g4:=x->(10/(4+x))^(1/2);
```

$$g4 := x \mapsto \sqrt{10}\,\sqrt{\frac{1}{4+x}} \quad \textbf{(10)}$$

```
> p0:=1.5;
```

$$p0 := 1.5 \quad \textbf{(11)}$$

```
> for n from 1 to 20 do
  p[n]:=evalf(g4(p0));
  err:=abs(p[n]-p0);
  if err>=10^(-4) then
  p0:=p[n];
  else
  break
  end if
  end do;
```

$$p_1 := 1.348399725$$
$$err := 0.151600275$$
$$p_2 := 1.367376372$$
$$err := 0.018976647$$
$$p_3 := 1.364957015$$
$$err := 0.002419357$$
$$p_4 := 1.365264748$$
$$err := 0.000307733$$
$$p_5 := 1.365225594$$
$$err := 0.000039154 \quad \textbf{(12)}$$

Illustration (e)

```
> restart;
> g5:=x->x-(x^3+4*x^2-10)/(3*x^2+8*x);
```

$$g5 := x \mapsto x - \frac{x^3 + 4x^2 - 10}{3x^2 + 8x} \tag{13}$$

```
> p0:=1.5;
```

$$p0 := 1.5 \tag{14}$$

```
> for n from 1 to 20 do
  p[n]:=evalf(g5(p0));
  err:=abs(p[n]-p0);
  if err>=10^(-4) then
  p0:=p[n];
  else
  break
  end if
  end do;
```

$$p_1 := 1.373333333$$
$$err := 0.126666667$$
$$p_2 := 1.365262015$$
$$err := 0.008071318$$
$$p_3 := 1.365230014$$
$$err := 0.000032001 \tag{15}$$